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DESIGN OPTIMIZATION FOR ROBUSTNESS USING QUADRATURE FACTORIAL MODELS

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ABSTRACT

This paper describes a robust optimization methodology for designs involving either complex simulations or actual experiments. The methodology adopts a new objective function which consists of an Expected Performance (*EP*) and a weighted Quality Index (*QI*). This definition enhances the measure of optimality and robustness. This paper introduces the Quadrature Factorial Experiment to estimate the expected performance and standard deviation. This technique greatly reduces the number of experiments and provides superior results for performance with significant interaction effects and nonlinear variations. The case study applies the proposed methodology to the design of helical gears with minimum peak-to-peak transmission error (PPTE) using the profile modification technique. The robust optimum shows a significant reduction of the expected PPTE compared with previous studies, while maintaining the insensitivity to profile errors, shaft misalignment, and load variation.

INTRODUCTION

All products are subject to the variations of raw material, manufacturing, and operational conditions. Quality designs must perform to specification throughout the intended product life despite these variations, and have excellent attributes such as low cost, weight, etc. Traditionally, engineers conducted sensitivity analysis after design optimization (Vanderplaats, 1984). It has become increasingly important to incorporate manufacturing and operational variations in the early design stage. This paper aims to develop a methodology to integrate the variations of design variables in the optimization process to achieve the "Robust Optimum" (Yu & Ishii, 1993).

Taguchi (1978) introduced the concept of *Parameter and Tolerance Design* to improve the quality of a product whose manufacturing process involves significant variability. Parameter design reduces performance deviation by reducing the sensitivity of an engineering design to sources of variations rather than controlling the sources. Taguchi treated tolerance design as the last resort to tighten the most sensitive variables and bring the performance deviation up to specification. The concept improves quality, but at a cost no higher than necessary.

Taguchi's approach is cost effective in improving product quality and has attracted public interests in the application of quality control at the design stage (Kackar, 1985; Hunter, 1985; Barker, 1986). There are plenty of successful applications in industry (Brendell, 1989; Wu, 1989; Sundaresan, 1989). Several researchers have extended Taguchi's method to optimization procedures. Tsai & Ragsdell (1988), Chang (1989), and d ntremont & Ragsdell (1988) adopted Taguchi's concept of quality-loss and cast the reliability of a product performance into a nonlinear programming procedure. Their goal is to minimize performance variability when the design is constrained to have target performance. In the case that both the performance and the deviation are design objectives, Sundaresan *et al.* (1989), Sandgren (1989), Eggert & Mayne (1990), and Yu & Ishii (1993) modified the objective function in the optimization procedure to seek the robust optimum.

On another front, several studies addressed the deficiency of the Taguchi's method in the selection of performance measure (Leon *et al.,* 1987; Tribus & Szonyi, 1989) and the experimental design of systems with potential interaction effects (Ryan, 1988). Box (1988) showed that one should not attempt a single criterion, such as Taguchi's signal-to-noise ratio, for all applications. He recommended using exploratory data analysis and evolutionary operation to identify the best transformation. Montgomery (1991) pointed out that Taguchi's approach to experimental design is weak in dealing with potential interactions between controllable factors and that his method of data analysis may confound location and dispersion effects. D'Errico & Zaino (1988) demonstrated a mathematical explanation of the Taguchi method of tolerance design, but showed that Taguchi's results are not optimal. They proposed a modified procedure for statistical tolerancing with better solutions. Few studies address the nonlinearity of performance deviation and variation correlation. Yu & Ishii (1994) introduced the concept of the Manufacturing Variation Patterns

(*MVP*) to characterize the coupled variations of design variables and determine the constraint uncertainty.

This paper proposes the robust optimization procedure for system involving experiments or complicated simulations. Section 2 introduces our new definition of objective function consisted of an Expected Performance (*EP*) and a weighted Quality Index (*QI*), which leads to the robust optimum. The methodology adapts the Fractional Quadrature Factorial (Yu, 1994) to estimate the expected performance and standard deviation. This technique greatly reduces the number of experiments and provides superior results for performance with significant interaction effects and nonlinear variations. Section 3 applies the proposed methodology to the design of helical gears with minimum peak-to-peak transmission errors. Conclusions and future work comprise section 4.

ROBUST DESIGN OPTIMIZATION

Background

Conventional optimization minimizes the nominal value of the objective function and overlooks the deviation of the function due to manufacturing and operation errors. Figure 1 highlights the difference between the conventional peak optimum *P* and the robust optimum *R*. For simplicity, the objective *y* is assumed to be a function of a single parameter *x*. The concept also applies to designs with multiple variables. Parameter *x* often contains a statistical variation due to manufacturing errors. Conventional optimizations believe that point *P* is the design optimum. However, *x* will normally distribute between P_h and P_l for a mass production process. The design might end up with an objective value as worse as P_l due to the variation of *x*. Naturally, one can tighten the tolerance of *x* to control the deviation of the objective. However, smaller tolerance will result in a higher controlling cost. In contrast, if we target the design at point *R*, the deviation of *y* becomes significantly smaller with the same distribution of *x*. In comparison with the design *P*, the design *R* is more obust". Although the nominal objective of point *R* may not be the minimum, the overall quality is better. Robust optimization seeks designs which are close to the peak optimum but insensitive to the variations of control parameters.

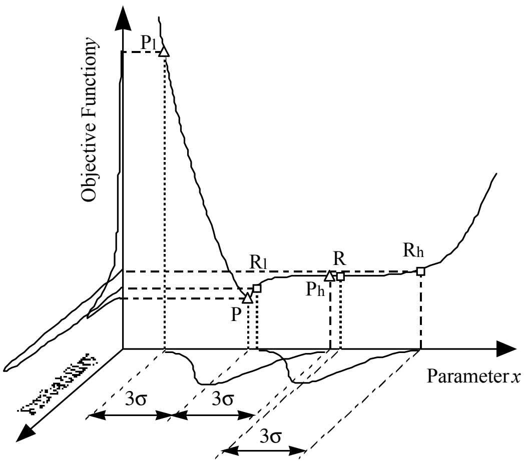


Figure 1. Peak optimum P and robust optimum R

Probabilistic Optimization

Conventional constrained optimizations minimize the nominal performance and take the following formulations:

Minimize	y(X)	(1)
Subject to	$g_j(X) \leq 0$ for j=1,2,...,J	(2)
	$h_k(X) = 0$ for n=1,2,...,K	(3)
	$X=(x_1, x_2,...., x_n)^T$	(4)

The design vector *X* is subject to inequality constraints $g_j(X)$ and equality constraints $h_k(X)$. x_i stand for the design variables, such as geometry, material, and manufacturing parameters, and often inherit variations. The objective $y(X)$ is a function of x_i and will thus have a statistical distribution. The constraints contain uncertainty as well due to the variations of design variables.

Probabilistic optimization (Siddall, 1984) takes into account of the uncertainties of control variables and constraints, and minimizes the expected value of the objective function. This concept remodels the formulations as follows:

Minimize $$\mu_y = E[y(X)] = \int_{X^-}^{X^+} y(X) \cdot p(X) \cdot dX \quad (5)$$

Subject to $Probability[g_1(X) \leq 0 \cap .. \cap g_J(X) \leq 0)] \geq P_f$ (6)

$E[h_k(X)] = 0$ for $k=1,2,...,K$ (7)

$X^- \leq X \leq X^+$

where p(X) is the joint probability function of X

P_f is the required probability of a feasible design

To satisfy the inequality constraints, the joint probability of the feasibility must be larger than a specified probability P_f which represents the rejection rate of the design. Probability optimization requires the joint probability density function $p(X)$ which is usually unknown. Even if the joint probability function is given, the evaluation of the expected value will be computationally intensive.

In addition, probabilistic optimization solely optimizes the expected value of response, which will not guarantee a robust design. Figure 2 gives an example of two designs with the same expected performance. However, Design 1 has a better quality than Design 2 because of the smaller performance deviation. Thus, robust design optimization should optimize expected value as well as performance deviation.

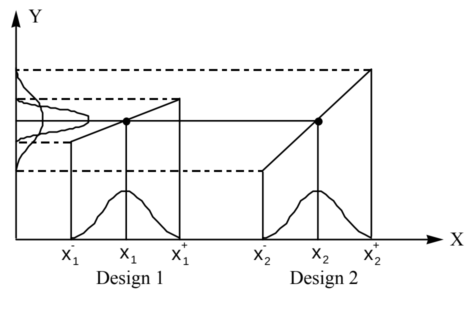


Figure 2. Designs of the same expected value but different variations

Objective Function for Design Robustness

One way to include performance deviation to design optimization is to modify design objective. Taguchi proposed the Signal-to-Noise (*S/N*) Ratios as performance criteria and claimed that the transformation of the *S/N* ratio separates the dependence between the mean and the standard deviation which facilitates the process of parameter design. However, Leon *et al.* (1987) pointed out that *S/N* is appropriate only when the deviation is proportional to the performance mean. Box (1988) also showed that *S/N* ratio confounds location and dispersion, and can be extremely inefficient.

Taguchi optimized the *S/N* ratio and did not consider the actual mean and deviation of the performance which may result in a solution far away from the peak optimum. Sandgren (1989) described the design process with a tree structure and included the design sensitivity to uncertainty parameter in multi-objective nonlinear goal programming. He assigned the weighting factors according to the priority ranking of each goal constraint. In contrast, Sundaresan *et al.* (1989) used Taguchi's orthogonal array and proposed a weighted sum of the center response and the sensitivity index (*SI)* as the design objective to seek the Statistical Optimum.

$$F(X)=\alpha * L_c+(1-\alpha)*SI \tag{8}$$

$$SI=\sqrt{\frac{1}{m}\sum_{i=1}^{m}(L_i-L_c)^2} \tag{9}$$

where α is the weighting factor, $0 \le \alpha \le 1$
m is the number of trials in the orthogonal array
L_c is the center response
L_i is the response in the orthogonal array

If the response variations are linear, L_c and SI will estimate the performance mean and standard deviation respectively. However, if nonlinearity becomes a concern, the actual meanings of L_c and SI become obscure.

Eggert & Mayne (1990) suggested another objective that consists of a weighted sum of the expected mean μ_y and the standard deviation σ_y of the performance *y*:

$$F(X)=\omega * \mu_y + \gamma * \sigma_y \tag{10}$$

However, the exact evaluations of expected means and variances will be very computationally intensive. Moreover, the actual joint probability function is usually not available. The selection of the weighting factors, ω and γ, are somewhat arbitrary, which makes the formulation lack physical meaning. Yu & Ishii (1993) advocated another form of the objective function based on the concept of statistical worst case to seek the Robust Optimum:

$$F(X)=\mu_y+\beta*\sigma_y \approx EP+\beta*QI \tag{11}$$

where Expected Performance (*EP*) is an estimate of expected mean
Quality Index (*QI*) is an estimate of performance deviation
β is the quality coefficient

The objective function consists of two components. *EP* leads to a probabilistic optimum in the optimization procedure, and *QI* ensures design robustness. Yu and Ishii used the Quadrature Factorial Design to estimate mean and performance deviation. Section 2.5 will give the details of the evaluation of *EP* and *QI*. The quality coefficient β in Equation (11) serves in two ways:

(1) If the performance y has a normal distribution, the objective function F(X) represents the statistical worst case response at the confidence level corresponding to β. For instance, if the original objective is to minimize *y(X)*, Figure 3 shows that F(X) will stand for the worst case of *y* at the confidence of 97.7% by selecting β=2. We can control the reject probability of the design using the appropriate β.

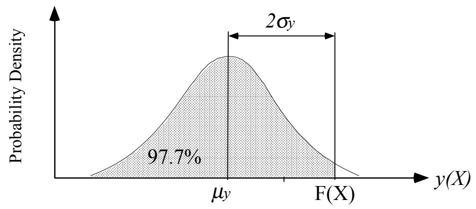


Figure 3. Illustration of the definition of the objective function

(2) If the distribution of Y is far from normal, the selection of β controls the design focus between expected performance and design quality. Higher weighting of *QI* gives designs with a smaller performance deviation, while decreased weighting will shift the design to a probabilistic optimum.

STATISTICAL EXPERIMENTAL DESIGN

Factorial experiment (Montgomery 1991) is a statistical technique that investigates the effects of two or more factors (design variables) by conducting the experiments at all possible combinations of the levels of the factors. The experimental size, however, multiplies quickly. Fractional factorial designs use only a fraction of the experiments to investigate the significant effects. The main factors and interaction effects are confounded (or mixed) with higher order interactions in a fractional factorial design. However, these higher order interactions are often negligible in practice, which enables us to get an accurate prediction in spite of the reduced number of trials. The bottom line is to design a fractional factorial so the significant effects will not confound with each other.

Fractional Quadrature Factorial Experiment

Two-level factorial works well if the effects are linear over the variation range. However, the accuracy becomes doubtful if nonlinearity is present. Three and higher level factorial designs give some improvement but result in a large and complex design. Instead of using 3-level FFE, we adopt the 2-level FFE augmented with center points (2FFEC) to cope with the possible nonlinearity, while at the same time keeping the size and complexity low. The augmentation of center points enables us to identify the pure quadratic effects of the model. If the curvature effect is significant, one needs to include the quadratic terms in the regression model. A statistical analysis program, such as

JMP (1989) will easily provide us with the model of the response surface as shown in Equation. (12).

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i<j} \sum \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \varepsilon \tag{12}$$

The theoretical evaluations of the mean and the variance of performance *y(x)* are as follows:

$$E[y(X)] = \mu_y = \int_{-\infty}^{+\infty} y(X) \cdot p(X) \cdot dX \tag{13}$$

$$\operatorname{var}[y(X)] = \sigma_y^2 = \int_{-\infty}^{+\infty} (y(X) - E(y))^2 \cdot p(X) \cdot dX \tag{14}$$

where $X = [x_1, \ldots, x_n]^T$
$p(X)$ = joint probability density function of X.

The evaluations are very expensive particularly for complex simulation models and designs involved with actual experiments. Moreover, the joint probability functions are usually not available. Numerical integration, such as Gaussian integration, uses a weighted sum of a finite number of response to estimate the expected value.

$$E[y(x)] \approx \sum_{i=1}^{n} w_i \cdot y(x_i) \tag{15}$$

$$\operatorname{var}[y(x)] \approx \sum_{i=1}^{n} w_i \cdot (y(x_i) - E(y))^2 \tag{16}$$

Taguchi (1979) adopted the similar concept and proposed a uniform weighting for variables with normal distribution. He chose $\mu_i \pm \sigma_i$ as high and low levels for a 2-level design, and μ_i and $\mu_i \pm \sigma_i (3/2)^{0.5}$ as the center, high and low levels respectively for a 3-level design. This method works well for the estimation of expected mean. The estimation of variance, however, becomes poor if y contains quadratic effects such as the regression model of Equation (12).

D rrico & Zaino (1988) presented an alternative method using the Gauss-Hermite quadrature integration (Engels, 1980). This approach provides better results particularly for nonlinear cases. Figure 4 illustrates the selection of levels and the corresponding weighting for normal and uniform variables. The two-point method will provide accurate estimate of the variance of linear function, while the three-point method is good for quadratic functions.

If x_i are independent variables, the joint probability density function can be represented as follows:

$$p(X) = p(x_1) \cdot p(x_2) \cdot \ldots \cdot p(x_n) \tag{17}$$

where $p(x_i)$ is the probability function of x_i.

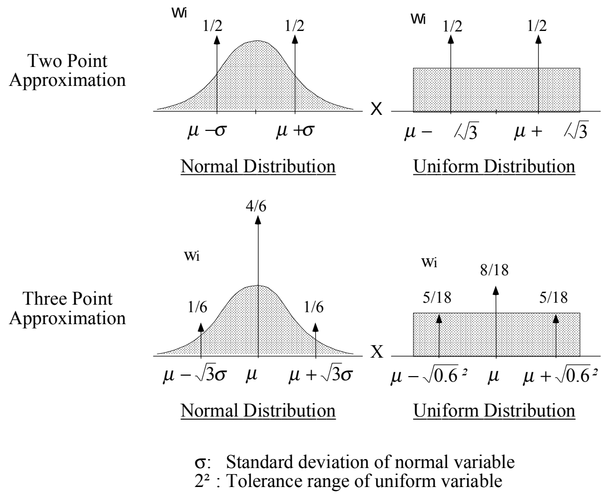


Figure 4. Selections of levels and weightings of normal and uniform variables

Quadrature factorial design selects the quadrature points as experimental levels to estimate expected mean and variance. The total weighting for each treatment (variable combination) is equal to the multiplication of the corresponding weighting of the individual level.

$$W_i = w_1 * w_2 \ldots * w_n \tag{18}$$

where w_i is the corresponding weighting of variable x_i at the assigned level

In practice, the performance nonlinearity within the variation range is moderate and can be characterized with a semi-quadratic regression model.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \underset{\substack{\text{significant}\\\text{interactions}}}{\sum\sum} \beta_{ij} x_i x_j + \beta_{11} \underset{\substack{\text{significant}\\\text{quadratic terms}}}{\sum} x_i^2 + \varepsilon \tag{19}$$

The center point is used to estimate the combining quadratic effect β_{11}.

Fractional Quadrature Factorial adapts the three-point quadrature method to the 2FFEC. The design defines the high and low levels as $\mu_i \pm \sqrt{3}\sigma_i$ for normal variables and $\mu_i \pm \sqrt{0.6}\Delta_i$ for uniform variables, and applies the least square fit of the fractional factorial data to obtain the local regression model as shown in Equation 19. The model generates the missing responses and expands the 2^{k-p} fractional factorial data to a contrived 3^k factorial as illustrated in Figure 5. The substitutions of the contrived 3^k factorial to Equations (20) and (21) produce the expected performance and the quality index.

$$E[y(X)] \approx EP \approx \sum_{i=1}^{N} W_i \cdot y_i \tag{20}$$

$$\sqrt{Var[y(X)]} \approx QI \approx \left[\sum_{i=1}^{N} W_i \cdot (y_i - E(y))^2 \right]^{1/2} \tag{21}$$

where $N = 3^n$

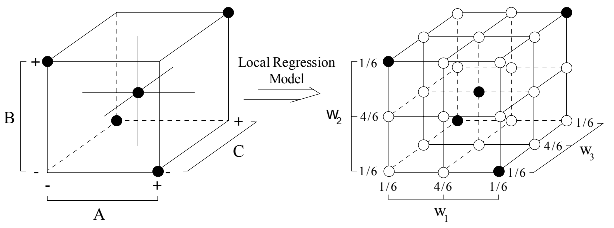
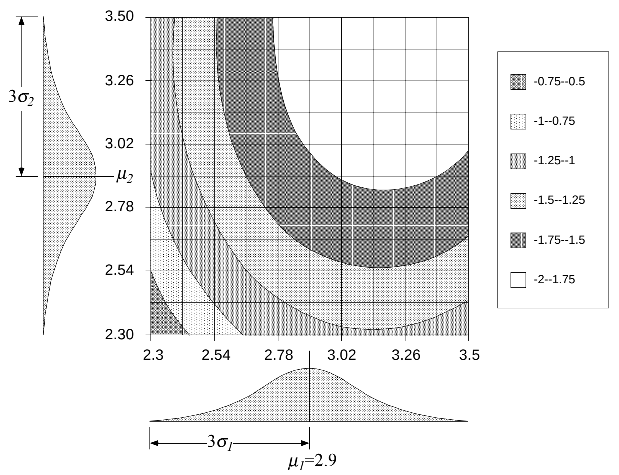


Figure 5. Contrived 3^k factorial data from 2^{k-p} FFEC

Numerical Example

Figure 6 shows a numerical example of a typical quadratic response surface within the $\pm 3\sigma$ tolerance range. Assuming x_i are normal variables, the expected mean of $f(x_1, x_2)$ is -1.6366 and the standard deviation is 0.1767 via simulations using 1681 function evaluations, a sufficiently large number to give precise estimates. Figure 7 shows the estimation results. All four methods give very accurate estimates of expected mean. However, the Fractional Quadrature Factorial provides a superior estimate of variance with a much smaller number of experiments.



$f(x_1, x_2) = 12.6 - 2.9*x_2 + 0.5*x_2^2 - 5.8*x_1 + x_1^2 - 0.2*x_1*x_2$

Figure 6. Sample quadratic response surface within the tolerance range

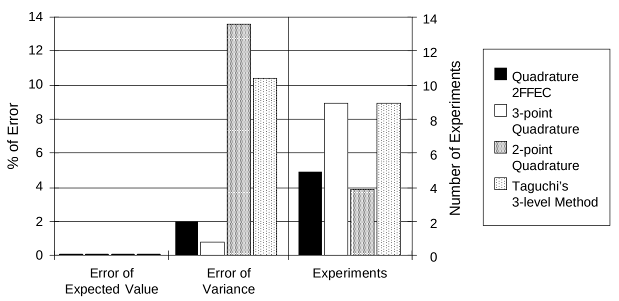


Figure 7. Estimation results of expected mean and variance

APPLICATION: HELICAL GEARS WITH MINIMUM TRANSMISSION ERRORS

Background

Welbourn (1979) defined the transmission error as he difference between the actual position of the output gear and the position it would occupy if the gear drive is perfect (infinite stiffness and conjugate teeth)." Manufacturing errors of tooth profile and shape deformations due to gear load, when coupled with the shaft misalignment, increase the transmission error drastically. Gear profile modification has been an effective technique to reduce the peak to peak transmission error (PPTE). The modifications enable the unloading of one mating tooth pair when the second pair makes initial contact, which lessens the sudden increase and decrease of mesh stiffness and reduces the variation of transmission error. However, the improvement is sensitive to manufacturing errors and load variations (Sundaresan, 1992). Our study seeks the robust optimum modification which has the least expected PPTE, while the performance is less sensitive to profile errors, shaft misalignment, and load variations.

Figure 8 illustrates the concept of profile modification. This study uses the Load Distribution Program (LDP, 1991) to predict the PPTE. The LDP is a computer program for predicting the load distribution across the zone of contact for a single pair of spur or helical gears. The method assumes the load distribution to be a function of the elasticity of the gear system and errors or modifications on the gear teeth. Lowe (1985) conducted experiments for spur gears, and showed that the predicting load distribution of LDP correlates well with the measured mesh deflection.

LDP characterizes the modification with two parameters: 1) the starting roll angle φ of the modification and 2) the amount of tip relief δ_T of the gear teeth.

$$\text{Starting Roll Angle } \varphi = \sqrt{\left(\frac{R_s}{R_b}\right)^2 - 1} \qquad (22)$$

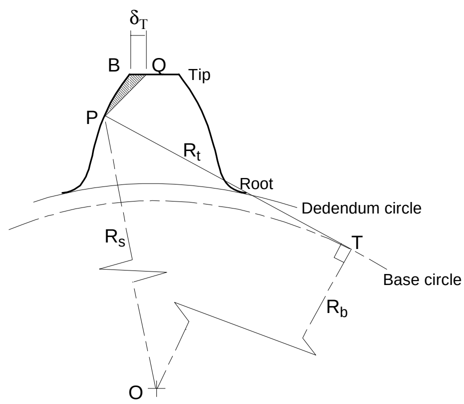


Figure 8. Gear profile modification

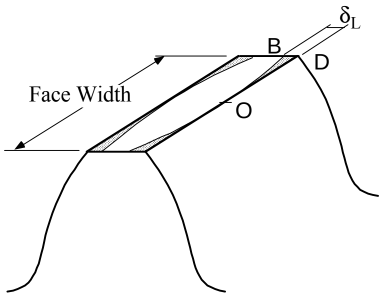


Figure 9. Parabolic lead modification

TABLE 1. HELICAL GEAR GEOMETRY
(SUNDARESAN, 1992)

GEOMETRIC PARAMETERS	Pinion	Gear
Transmitted torque (in-lbs)	750.0	
Center Distance (inch)	2.7953	
Normal diametral pitch (1/inch)	12.05	
Normal pressure angle (degree)	16.0	
Helix angle (degree)	30.0	
Profile contact ratio	2.02	
Face contact ratio	1.13	
Total contact ratio	3.15	
Number of teeth	18	41
Face width (inch)	0.6	0.6
Outer diameter (inch)	2.0663	4.0057
Root diameter (inch)	1.5517	3.4911
Roll angle @ pitch circle	16.70˚	16.70˚
Roll angle @ outer circle	44.09˚	22.44˚
Roll angle @ SAP*	3.63˚	4.68˚

*SAP: The Start Radius of the Active Profile

The modification can be either linear or parabolic. Figure 9 shows a parabolic lead modification with no modification at the center of the gear face and equal modifications at both end faces. The total amount of profile modification will be the sum of the lead modification and the tip relief.

Figure 10 shows the contour plot of the peak-to-peak transmission error of the helical gears and the operating conditions listed in Table 1. The plot assumes the parabolic profile modifications with equal amounts of tip relief on the gear and the pinion and a fixed 0.0003 inch parabolic lead modification. The starting roll angle of modification on the gear is varied proportionately with the pinion. The lowest contour, 12.5 micro inches, encloses the optimum profile modification that minimizes the PPTE. However, the design is liable to manufacturing and operational variations. The manufacturing errors of the tip relief and the starting roll angle are approximately 0.00015 inch and 1.5 degrees respectively, which induce a significant variation of the performance.

Sundaresan *et al.* (1989, 1992) adopted Taguchi's orthogonal array and the Sensitivity Index (Equation 8) in the optimization process to achieve a statistical optimum. The statistical optimum shows a slight increase of the target PPTE but greatly improves the worst case performance, which provides a better overall quality than the conventional peak optimum. However, Sundaresan methodology does not consider interaction effects and the significant nonlinearity of performance deviation. The nonlinearity of the performance deviation drastically differs the target and the expected performance. This study applies the proposed objective function (Equation 11) and the Fractional Quadrature Factorial to seek the Robust Optimum.

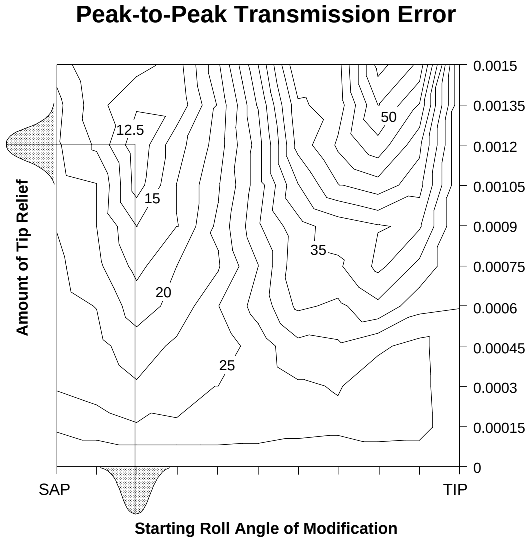


Figure 10. Contour plot of the PPTE for the helical gears in Table 1

<u>Robust Optimization for Gear Profile Modification</u>

This study optimizes the peak-to-peak transmission error of helical gears using the gear profile modification. Two case studies based on two different variation models show the flexibility and effectiveness of our methodology.

Case 1: Machined gears with small torque variation
Case 2: Machined gears with large torque variation

This study adopts the optimization program OPTPAK (1990) and selects the Broydon-Fletcher-Goldfarb-Shanno (BFGS) variable metric method (Vanderplaats, 1994) during the search of optimum.

<u>Case Study 1: Machined Gears with Small Torque Variation</u>. This study assumes independent variables. The design ranges of the variables are as follows:

1) Starting roll angle of modification on the pinion φ_p from the Start radius of Active Profile (SAP) to the tip of the tooth.
2) Starting roll angle of modification on the gear φ_g from SAP to the tip of the tooth.
3) Amount of tip relief on the pinion tooth δ_{Tp} from 0.0 to 0.0015".
4) Amount of tip relief on the gear tooth δ_{Tg} from 0.0 to 0.0017".
5) Amount of lead modification δ_{Lg} at both end faces of the pinion tooth from 0.0 to 0.0005". The lead modification was parabolic with zero at the center of the

face width. The amount of modification at both end faces of the pinion tooth was assumed to be equal and the gear tooth was unmodified in the lead direction.

The optimization considers four variations:

1) Variation of 0.00015 inch in the parabolic tip relief of pinion and gear
2) Variation of 1.5 degrees in the starting roll angle of pinion and gear
3) Torque (T) variation of 200 lb-ins
4) Shaft misalignment δ_S of 0.0005 inch per inch of face width.

Table 2 shows the design of experiment. Unlike full factorial design which needs 64 experiments, this design uses only nine experiments. The design allows us to investigate all the six main effects if the interactions are negligible.

TABLE 2. FRACTIONAL FACTORIAL ARRAY #1 OF THE GEAR DESIGN EXAMPLE
(Six main effects)

		φ_p	δ_{Tp}	φ_g	δ_{Tg}	T	δ_S	
2^{6-3}+1 (III)		**A** (BD) (CE)	**B** (AD) (CF)	**C** (AE) (BF)	**D** (AB) (EF)	**E** (AC) (DF)	**F** (BC) (DE)	(AF) (BE) (CD)
Experi-	*R1*	+	-	-	-	-	+	+
ment	*R2*	-	+	-	-	+	-	+
No.	*R3*	-	-	+	+	-	-	+
	R4	+	+	+	+	+	+	+
	R5	+	+	-	+	-	-	-
	R6	+	-	+	-	+	-	-
	R7	-	+	+	-	-	+	-
	R8	-	-	-	+	+	+	-
	R9	0	0	0	0	0	0	0

Note: "+" represents the high level , "-" represents low level, and "0" represents the middle level.

The variations of the PPTE due to the errors of the tip relief and the starting roll angle are significantly nonlinear as shown in Figure 9. The augmentation of the center point in our experimental design estimates the combining effect of the significant quadratic terms. The regression model of the PPTE within the variations of parameters takes the following form:

$$PPTE=\beta_0+\beta_1*A+\beta_2*B+\beta_3*C+\beta_4 D+\beta_5*E+\beta_6*F+\beta_7*(A^2+B^2+C^2+D^2) \quad (23)$$

Our experimental design adopts the Quadrature Factorial which selects the quadrature points as experimental levels. This study assumes normal variables, and the given variations represent three times of the standard deviations. The corresponding high and low levels are $\mu_i \pm \sqrt{3}\sigma_i$ of each variable.

The Quality Coefficient β in the objective function (Equation 11) is set at 2.0. The objective will represent the highest PPTE of each design at the probability of 97.7% due to the manufacturing and operational errors. Table 3 shows the optimization results with the values of the design variables for the peak optimum, the statistical optimum (Sundaresan, 1992), and the robust optimum.

The peak optimization leads to an inferior design since the target PPTE does not represent the actual performance of the design. For instance, the robust optimum though targets at a higher PPTE, in fact, holds the lowest expected PPTE. The actual performance distributes due to manufacturing and operational variations. One should consider both the expected response and the performance deviation. Our robust optimum outperforms the statistical optimum, and improves the expected PPTE by 45% and the performance deviation by 25% compared with the peak optimum. Figure 11 shows the optimization results graphically and indicates the worst PPTE of each design.

TABLE 3. COMPARISON OF THREE OPTIMUMS IN CASE-1

PARAMETERS		Peak	Statistica l	Robust
φ_p	(deg.)	11.45	16.57	14.41
φ_g	(deg.)	13.01	12.81	13.01
δ_{Tp}	(10^{-3} in)	1.175	1.053	1.051
δ_{Tg}	(10^{-3} in)	1.341	1.144	1.059
δ_L	(10^{-3} in)	0.283	0.000	0.001
Target PPTE	(μ in)	5.84	10.00	10.61
Exp. PPTE (*EP*)	(μ in)	11.39	11.42	6.31
Dev. of PPTE (*QI*)	(μ in)	5.88	3.60	4.42
Objective Function (*EP*+2**QI*)	(μ in)	23.15	18.63	15.14

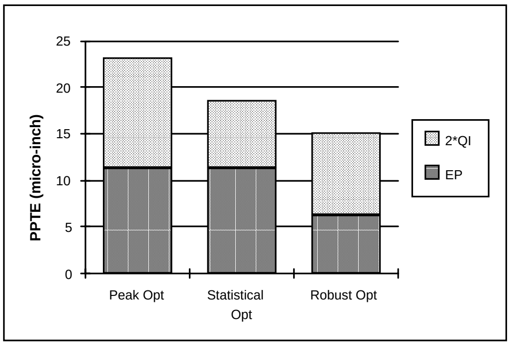


Figure 11. Optimization results of Case-1

<u>Case Study 2: Machined Gears with Large Torque Variation.</u> Sundaresan (1992) indicated that the misalignment has little effect on the total performance deviation; however, the interaction between tip relief and loading torque is significant. When the variation of loading torque increases, the interaction effect becomes notable. The variation model should thus include the significant interaction terms. To investigate the interaction effects using the same number of experiments, this case study considers only three variations:

1) Variation of 0.00015 inch in the parabolic tip relief δ_T of pinion and gear
2) Variation of 1.5 degrees in the starting roll angle φ of pinion and gear
3) Torque (T) variation of 200 lb-ins

We select (2^{5-2}) 2FFEC which allows the estimation of the interaction effects of tip relief and torque. Table 4 shows the arrangement of the *L8* experimental array. The corresponding regression model of the PPTE within the variations is as follows:

$$PPTE=\beta_0+\beta_1*A+\beta_2*B+\beta_3*C+\beta_4 D+\beta_5*E+\beta_6*BE +\beta_7*DE+\beta_8*(A^2+B^2+C^2+D^2) \quad (24)$$

TABLE 4. FRACTIONAL FACTORIAL ARRAY #2 OF THE GEAR DESIGN EXAMPLE
(Five main and two interaction effects)

		φ_p	δ_{Tp}	φ_g	δ_{Tg}	T	$\delta_{Tg}\times T$	$\delta_{Tp}\times T$
2^{5-2}+1 (III)		**A** (BD) (CE)	**B** (AD)	**C** (AE)	**D** (AB)	**E** (AC)	**DE** (BC)	**BE** (CD)
Experiment No.	*R1*	+	-	-	-	-	+	+
	R2	-	+	-	-	+	-	+
	R3	-	-	+	+	-	-	+
	R4	+	+	+	+	+	+	+
	R5	+	+	-	+	-	-	-
	R6	+	-	+	-	+	-	-
	R7	-	+	+	-	-	+	-
	R8	-	-	-	+	+	+	-
	R9	0	0	0	0	0	0	0

Table 5 shows the optimization results and the values of the design variables of the peak and robust optima. The robust optimum improves the expected PPTE by 15% and the performance deviation by 34% compared with the peak optimum. Figure 12 shows that the statistical worst case of PPTE could be as high as 22.3 micro-inches for the peak optimum while only 16.8 micro-inches for the robust optimum.

TABLE 5. COMPARISON OF THE PEAK AND ROBUST OPTIMUMS IN CASE-2

PARAMETERS		Peak	Robust
φ_p	(deg.)	11.45	14.41
φ_g	(deg.)	13.01	13.03
δ_{Tp}	(10^{-3} in)	1.175	1.051
δ_{Tg}	(10^{-3} in)	1.341	1.058
δ_L	(10^{-3} in)	0.283	0.000
Exp. PPTE (*EP*)	(μ in)	10.76	9.17
Dev. of PPTE (*QI*)	(μ in)	5.75	3.81
Objective Function (*EP*+2**QI*)	(μ in)	22.26	16.80

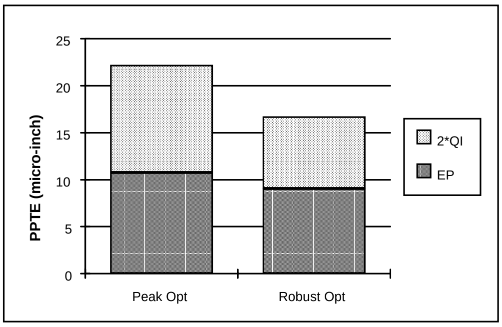


Figure 12. Optimization results of Case-2

CONCLUSION

This paper proposed a methodology that incorporates manufacturing errors and operational variations to seek the robust optimum for designs involved with either complex simulations or actual experiments. We introduced a new objective function which consisted of an Expected Performance (*EP*) and a weighted Quality Index (*QI*). This definition enhances the measure of optimality and robustness. The procedure adapts the Fractional Quadrature Factorial to estimate the expected performance and the standard deviation. This technique greatly reduces the number of experiments and provides superior estimation of design robustness even if the system contains significant interaction effects and nonlinear variations.

The paper presented an application of the proposed methodology to the design of helical gears with minimum peak-to-peak transmission error. The case study involved different operational conditions which led to two variation models. The proposed scheme readily accommodated the interaction effects and generated the corresponding robust optimum which excelled the previous study.

The current quadrature factorial assumed independent normal or uniform variables. However, there are designs with asymmetric distributions due to the transformation from manufacturing process parameters to design variables. The transformation could be nonlinear and induces complicated distributions of design variables. Future study is to address the transformation between manufacturing and design variables, and derive the corresponding quadrature points. Future application also includes the design of plastic gears where variation correlation is significant due to material shrinkage.

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利用高斯點因子試驗法進行穩健最佳化設計

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摘要

本論文提出一套設計方法，在最佳化設計的過程中結合製造與負載對設計參數造成的變異性，降低輸出對參數變異的敏感度，以尋求穩健化的最佳設計，作者研究參數的交互作用與非線性造成的變異特性，並採用輸出期望值與標準差的權重和做為最佳化的目標函數，而在參數的估計上，則結合統計實驗計劃與高斯積分法，即使輸入與輸出參數的變異具非線性，此方法仍能迅速準確地估計出設計輸出之期望性能與穩健性，本文亦包含此設計法則應用於精密齒輪設計以降低傳動損耗的模擬案例。

Keywords: Robust Design, Factorial Experiment, Taguchi Method, Statistical Optimization, Gear Design