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# DESIGN FOR ROBUSTNESS USING MANUFACTURING VARIATION PATTERNS

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## ABSTRACT

This paper addressed the parameter variations due to interacting manufacturing errors and their impact to the design robustness and constraint activity. Manufacturing errors often affect design parameters with characteristic patterns which is crucial to the determination of the output distribution. This paper proposes the *Manufacturing Variation Pattern (MVP)* to represent this variation characteristic and investigates the effects of the level of correlation among design parameters. The extension of the concept of *MVP* to the design optimization leads to a feasible robust optimum which contains the superior expected performance and the least sensitivity to the manufacturing variations and constraint uncertainty. The design of molded gears with minimum peak-to-peak transmission error illustrates the application of the proposed algorithms. The characteristic of the manufacturing process of molded gears results in the correlation of dimensional parameters. Our design scheme readily accommodates the variation correlation and provides designs with significant reduction of the performance deviation due to manufacturing and operational errors.

## INTRODUCTION

Recent advances in quality engineering urge designers to consider deviations of design variables in the early stages of design. The deviations may result from uncertainties in the raw material, manufacturing, and operation conditions. Quality products should perform to specifications despite these variations and have excellent attributes such as low cost, high performance, etc. Traditionally, engineers conducted sensitivity analysis after design optimization. Taguchi (1978) introduced the concept of *parameter design* which educes deviation in performance by reducing the sensitivity of an engineering design to sources of variations rather than controlling the sources."

Chang (1989) and d'Entremont & Ragsdell (1988) adopted Taguchi's concept of quality-loss and cast the reliability of a product performance into a nonlinear programming procedure. Their goal is to minimize performance variability when the design is constrained to have target performance. In contrast, Sundaresan *et al.* (1991), Sandgren (1989), Eggert & Mayne (1990), and Yu & Ishii (1993) integrated design performance and deviation into the objective function of optimization procedure to seek the robust optimum.

On another front, Montgomery (1991) pointed out that Taguchi's approach to experimental design is weak in dealing with potential interactions between controllable factors and that his method of data analysis may confound location and dispersion effects. D'Errico & Zaino (1988) also criticized that Taguchi statistical tolerancing is not optimum, and proposed a modified solution using the Gaussian-Hermite quadrature integration.

Another challenge in robust design is dealing with constraint uncertainties. Design variables are subject to manufacturing variations which result in constraint uncertainties. Conventional actively constrained optimum may not be statistically feasible. Parkinson *et al.* (1990) advocated a two-step solution to modify the feasible region in constrained optimization. Sundaresan *et al.* (1993) compared the efficiency of three different methods which incorporate variations in constraints. Most of these studies emphasized the propagating errors of constraints using the simultaneous worst case of parameter variations, and fell short in addressing the nature of variations.

This paper focuses on parameter variations due to interacting manufacturing errors. Manufacturing errors often affect design variables with characteristic patterns. These atterns" of manufacturing errors are particularly important in net shape manufacturing, such as injection molding of gears. Here, the dimensional error is largely due to shrinkage which simultaneously affects multiple design variables. Other processes such as heat treatment distortion of transmission axles and grinding of spiral bevel gears affect the critical dimensions in a heavily coupled manner. One can no longer assume that variations on multiple design variables are independent. However, few studies address this issue of interdependency among variations on variables and constraints.

Our study introduces the concept of Manufacturing Variation Pattern *(MVP)* to characterize the coupled variations of design variables. The shape of the variation pattern depends on the distributions and the correlating levels of design variables. Different designs and different manufacture processes have their unique variation patterns. Yu & Ishii (1994) referred this process as atching the design to the manufacturing variation patterns." The performance variation within the pattern will determine the design robustness. This paper develops an algorithm to find the feasible robust optimum that matches the pattern. The design of the molded helical gear with minimum peak-to-peak transmission error serves as an example to illustrate the utility of the proposed methodology.

## REVIEW OF ROBUST OPTIMIZATION

### Robust Optimization Modeling

Conventional constrained optimizations minimize the nominal performance and take the following formulations:

Minimize $y(X)$ (1)

Subject to $g_j(X) \leq 0$ for j=1,2,...,J (2)

$h_k(X) = 0$ for k=1,2,...,K (3)

$X=(x_1, x_2,...., x_n)^T$ (4)

$x_i$ represent design variables, such as geometry, material, and manufacturing parameters, and often inherit variations. The objective $y(X)$ is a function of $x_i$ and will thus have a statistical distribution. The design vector $X$ is subject to inequality constraints $g_j(X)$ and equality constraints $h_k(X)$ which also contain uncertainty due to the variations of design variables.

Probabilistic optimization (Siddall, 1984) takes into account of the uncertainties of design variables, and minimizes the expected mean of the objective function.

Minimize $$\mu_y = E[y(X)] = \int_{X^-}^{X^+} y(X)\cdot p(X)\cdot dX \quad (5)$$

where $p(X)$ is the joint probability function of $X$

$\sigma_y$ is the standard deviation of $y$

However, probabilistic optimization will not guarantee a satisfactory robust design which requires not only a optimum expected performance but also a small performance deviation. Yu & Ishii (1993) suggested the summation of the expected performance and a weighted standard deviation as the object function to seek the robust optimum.

$$F(X) = \mu_y + \beta * \sigma_y \quad (6)$$

where $\beta$ is termed the Quality Coefficient

### Estimations of Expected Mean and Variance

The evaluation of the objective function in the robust optimization often requires the information of the expected means and the variances of performance. The exact evaluations involve an integration with the joint probability function of control variables, that will be very computationally intensive. Besides, the joint probability function is often unavailable. One alternative using Taylor expansion for independent variables takes the following formulations:

$$\mu_y \approx y(M) + \frac{1}{2}\sum_{i=1}^{n}\left(\left.\frac{\partial^2 y}{\partial x_i^2}\right|_M\right) * \sigma_i^2 \quad (7)$$

$$\sigma_y^2 \approx \sum_{i=1}^{n}\left(\left.\frac{\partial y}{\partial x_i}\right|_M\right)^2 * \sigma_i^2 \quad (8)$$

where $\sigma_i^2$ are the variance of $x_i$.

$$M = E[(x_1,....,x_n)] = (\mu_1, \mu_2, ..., \mu_n)$$

However, the approximations require a closed-form model of response. If the designs involve either complex simulations or experiments where simple system models are not available, the evaluations of response derivatives become impractical. Yu (1995) proposed the Fractional Quadrature Factorial Model which combine fractional factorial and Gaussian-Hermite integration to approximate response variations. Yu scheme selects the quadrature points of the variable distribution as the factorial levels, and applies a weighted sum of the fractional factorial set to estimate the expected response and the performance deviation. This method provides superior approximations even if the system contains significant nonlinearity effects, and can be readily adapted to the case of correlated variables.

$$\mu_y \approx EP = \sum_{j=1}^{N} W_j \cdot y_j \quad (9)$$

$$\sigma_y^2 \approx DI^2 = \sum_{j=1}^{N} W_j \cdot (y_j - E(y))^2 \quad (10)$$

where $N=3^n$

$W_j = w_1 * w_2 ... * w_n$

$w_i$ are the weightings of design variable $x_i$ at their respective levels

$EP$ (Expected Performance) is an estimate of expected mean

$DI$ (Deviation Index) is an estimate of the standard deviation of performance

## MANUFACTURING VARIATION PATTERNS

### Definition

Manufacturing errors often induce statistical scatter to design variables which may be independent or correlated. Most robust design schemes to date use the worst case region (WCR) to represent the variation space of design variables. The shape of the WCR of a two dimensional problem is a rectangle as shown in Figure 1(a). WCR specifies each variable by the individual confidence interval and does not take into account the joint distribution of the variables.

Consider two independent normal variables. Conventional worst case regions use the Bonferroni method (Rawlings, 1988). The overall confidence coefficient is the product of all the univariate confidence coefficients. Figure 1(a) shows that the intersection of two 97.5% univariate confidence intervals leads to a rectangular region with a simultaneous confidence coefficient of 0.95. However, it would be misleading if one interprets the rectangular intersection as a joint confidence region since some designs inside the pattern are unlikely parameter combinations at the probability of 95%.

The variation pattern of design variables should represent the possible combination of parameters at the specified probability. Worst case region lacks the actual meaning of statistical distribution. This paper proposes the concept of the *Manufacturing Variation Pattern* as follows.

Def. 1: Manufacturing Variation Pattern (*MVP*)

*Manufacturing Variation Pattern* is a set of designs that belong to the (1-$\alpha$)* 100% joint confidence region of the target design. *MVP(1-$\alpha$)* denotes the space of possible parameter combinations at the confidence coefficient of (1-$\alpha$) where $\alpha$ indicates the probability of the design outside the variation pattern.

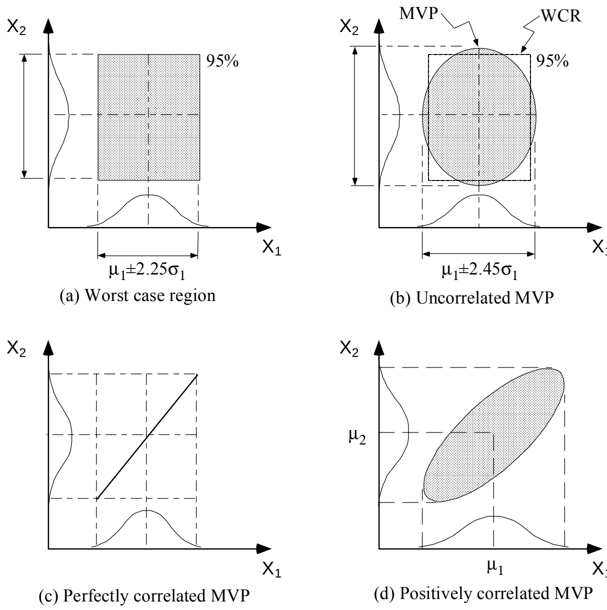



Figure 1. Worst case region vs. various Manufacturing Variation Patterns

The distributions of the variables determine the shape of *MVP*. *MVP* could be a rectangular solid ONLY if the distributions of the design variables are uniform. However, in many mass production environments, the variables assume normal distributions. The corresponding *MVP* will be an ellipsoid which will become a sphere with appropriate scaling of the variables. The confidence coefficient (1-$\alpha$) determines the size of ellipsoid. Figure 1(b) shows the *MVP(0.95)* of two independent normal variables. As one can see from the overlapped patterns, the covered areas of WCR and *MVP* are quite different even if the variables are independent.

Variation correlation changes the orthogonality of *MVP*. If the variations are perfectly correlated, one can identify a functional relationship between the variations of $x_i$. The corresponding *MVP* becomes a line or a curve as shown in Figure 1(c). Partial correlation between variables changes the *MVP* to an oblique ellipsoid. The slope of the axes of the ellipsoid shows the direction of correlation between parameters. Figure 1(d) shows a typical pattern for a positively correlated *MVP*. Design robustness and feasibility are directly related to the variation pattern of design variables. One should study the correlation among variables to select the correct patterns, since erroneous assumptions of the manufacturing variation patterns will lead to inferior designs.

**Manufacturing Variation Patterns for Typical Processes**

Orthogonal ellipsoid represents a typical variation pattern for conventional manufacturing processes such as lathe-turning, grinding, and milling. The distributions of the dimensions in the axis directions of machining processes are mostly normal and independent. Dimensions *w* and *D* of the lathe-turned shaft in Figure 2 are typical examples.

However, certain post-machining process, such as heat treatment, will change the independence among variables. For instance, dimensions *w* and *D* of the shaft in Figure 2 will distort after through hardening. The volume change due to the phase transformation of constituent correlates the dimensional distortions. However, interactions of thermal and transformation stresses further complicate the relationship. According to Ameen Rule (1940), dimensional changes resulting from temperature induced stresses will cause the shape of a component to become more spherical, which introduces a negative correlation between *w* and *D*.

Figure 3 presents an actual variation pattern of a heat treated spline shaft. The hardening processes consist of a furnace heating at 1540˚F and a solution quenching at 430˚F. The shaft is then tempered at 850˚F to reach Brinell hardness 400-444. The *MVP* clearly shows a negative correlation between the width and the spline diameter over pins.

The dimensional changes of injection molded and die casted parts have another correlation pattern. Volumetric shrinkage $v$ affects the dimensional changes of plastic part. If the material is homogeneous and the cooling and packing variations are negligible, the linear shrinkage rate will be homogeneous and approximately $v^{1/3}$. However, for more complex parts such as the plastic module housing in Figure 4, the variations of packing pressure, mold temperature, melt temperature, and the interaction of geometric features become significant. The perfect correlation among dimensions becomes obscure. Figure 5 shows the *MVP* of the actual production measurements of the plastic module housing. Dimension $x_1$ exhibits primarily linear shrinkage, and dimension $x_2$ exhibits both linear shrinkage and warpage. The oblique pattern shows a strong positive correlation between $x_1$ and $x_2$.

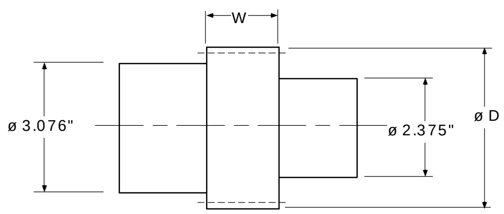
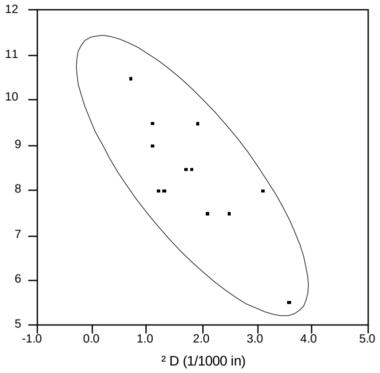



Figure 2. Example part of heat treated spline shaft
Number of teeth = 28, Diametral pitch = 8
Pressure angle = 25˚




| Variable | Mean | Std Dev | Correlation | Signif. Prob | Number |
|---|---|---|---|---|---|
| ΔD | 1.841667 | 0.869125 | -0.7896 | 0.0023 | 12 |
| ΔW | 8.333333 | 1.267304 | | | |

Figure 3. *MVP*(0.95) for *W* and *D* of the heat treated shaft

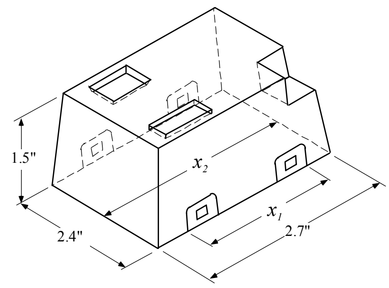
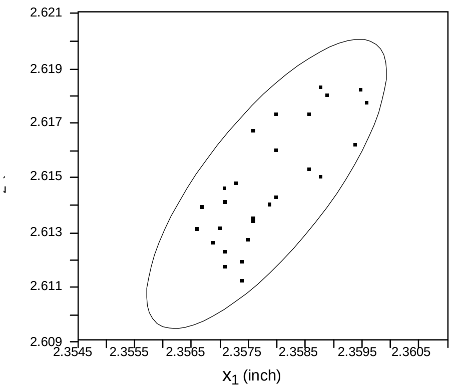



Figure 4. Plastic module housing




| Variable | Mean | Std Dev | Correlation | Signif. Prob | Number |
|---|---|---|---|---|---|
| $x_1$ | 2.357856 | 0.00088 | 0.770998 | 0.0000 | 27 |
| $x_2$ | 2.614711 | 0.00215 | | | |

Figure 5 *MVP*(0.95) for $x_1$ and $x_2$ of the injection molded module housing

## Derivation of Manufacturing Variation Patterns

This study applies the multivariate statistical techniques to derive the variation patterns. Typical mass production processes usually have well-established statistical data of variable distributions. Consider normal variables $x_i$ with means $\mu_i$ and variance-covariance $\sigma_{ij}$, the formulation for the $n$ dimensional ellipsoid of the *MVP(1-α)* is as follows:

$$(X-M)^T\Sigma^{-1}(X-M) \le \chi^2_{(n,\alpha)} \qquad (11)$$

where $X = [x_1,....,x_n]^T$

$M = E[x_1,....,x_n]^T = [\mu_1, \mu_2,..., \mu_n]^T$

$\sum$ is the variance-covariance matrix

$\chi^2_{(n,\alpha)}$ is the value of the chi-square value with $n$ degrees of freedom that leaves probability $\alpha$ in the upper tail.

The axes of the confidence ellipsoids lie in the directions of the eigenvectors, $e_i$, of $\sum$. The lengths of the principal axes are equal to $\sqrt{\chi^2_{(n,\alpha)}\lambda_i}$, where $\lambda_i$ are the eigenvalues of $\sum$.

If the variables $x_i$ are independent, the formulation can be simplified as follows:

$$\sum_{i=1}^{n}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2 \le \chi^2_{(n,\alpha)} \qquad (12)$$

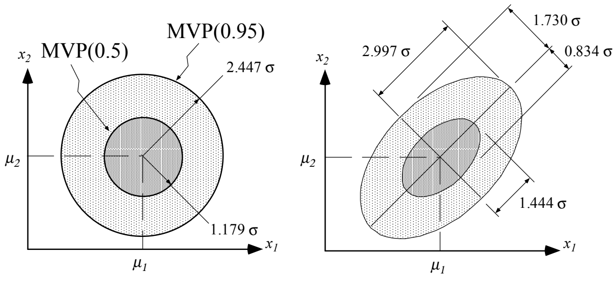
The pattern corresponds to an orthogonal ellipsoid with the lengths of the principal axes $\sqrt{\chi^2_{(n,\alpha)}}\sigma_i$. In practice, we use the sampling average $\overline{x_i}$ to estimate $\mu_i$, and the sampling standard deviation $s_i$ to estimate $\sigma_i$.

The correlation coefficient $r_{ik}$ measures the strength of the linear association between two variables, $x_i$ and $x_k$.

$$r_{ik} = \frac{s_{ik}}{\sqrt{s_i^2}\sqrt{s_k^2}} = \frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)(x_{kj}-\bar{x}_k)}{\sqrt{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}\sqrt{\sum_{j=1}^{n}(x_{kj}-\bar{x}_k)^2}} \qquad (13)$$

If $r$=0, it implies a lack of linear association between these two variables. Otherwise, the sign of $r$ indicates the direction of the association. If $r$ is close to 1 or -1, a linear relation may exist between these two variables. Figure 6 shows the *MVP*(0.50) and the *MVP*(0.95) of a bivariate normal example with common variance. The confidence coefficient (1-$\alpha$) determines the size, and the correlation coefficient $r_{12}$ affects the orthogonality ellipticity of the pattern.

For variables with some other distributions or nonlinear correlation, their *MVP*s will be much more complicated. However, the *central limit* effect of statistics suggests that the sampling distributions of many multivariate statistics are approximately normal, regardless of the form of the parent population. The variation patterns for other distributions will exhibit in a similar manner.




(a) $\sigma_1=\sigma_2=\sigma$, $r_{12}$=0 (b) $\sigma_1=\sigma_2=\sigma$, $r_{12}$=0.5

Figure 6. *MVP*(0.50) and *MVP*(0.95) of a bivariate normal example

## MANUFACTURING VARIATION PATTERNS AND CONSTRAINED ROBUST OPTIMIZATION

### Design Robustness

The design robustness is a function of expected mean and performance deviation which are greatly affected by the variation pattern of design variables. The proposed methodology combines the concept of *MVP* and the Quadrature Factorial Model to evaluate the objective function. The scheme better estimates the design robustness which ensures a robust optimum. The *MVPs* are orthogonal ellipsoids for independent normal variables; thus we can directly apply equations (9) and (10) to evaluate *EP* and *DI*. However, the *MVP* of correlated variables will become oblique ellipsoid. One need to apply transformation techniques to decouple the variables before the selection of quadrature factorial. A transformation technique can decouple the variables to identify the appropriate quadrature factorial.

The transformation between these coordinates is:

$$X = M + [e_1, e_2, \cdots, e_n]Z \quad (14)$$

where $X = [x_1,....,x_n]^T$, $Z = [z_1,....,z_n]^T$, $M = [\mu_1, \mu_2,..., \mu_n]^T$
$e_i$ are the eigenvectors of the variance-covariance matrix $\sum$

The factorial experiments should select the quadrature points along the *Z* axes as shown in Figure 7. The corresponding high and low levels in terms of *Z* coordinates are $\pm\sqrt{3\lambda_i}$, where $\lambda_i$ are the eigenvalues of $\sum$.

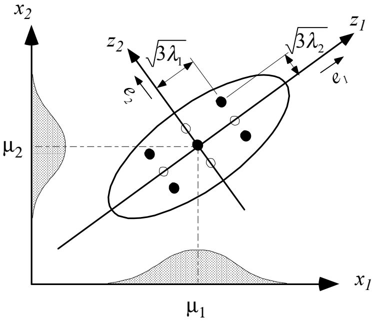



Figure 7. Transformation of factorial experiments to decouple correlated *MVP*

### Constraint Uncertainty

Manufacturing errors introduce deviations to design variables and propagate to design constraints. Robust optimization uses statistic techniques to redefine equality and inequality constraints. A design satisfies the equality constraint $h_k(X)$ if the expected mean of $h_k(X)$ is equal to zero. However, the satisfaction of inequality constraints will present a probability. A constrained optimum should be statistically feasible regardless of constraint uncertainties. This paper defines the robust feasibility as no constraint violation within the *MVP*.

Conventional peak constrained optimum may contain a large portion of unsatisfactory designs due to uncertainties (Figure 8a). One alternative is to move the design to the worst case actively constrained point (Sundaresan *et al*, 1993) as shown in Figure 8(b). However, the worst case region does not capture the actual distribution of design variables. The constrained solution may not be the true optimum as we examine the overlay *MVP*s in Figure 8(b). The worst case actively constrained design may be over or under constrained depending on the actual variation patterns, which leads to inferior designs if the performance is sensitive to the variations.

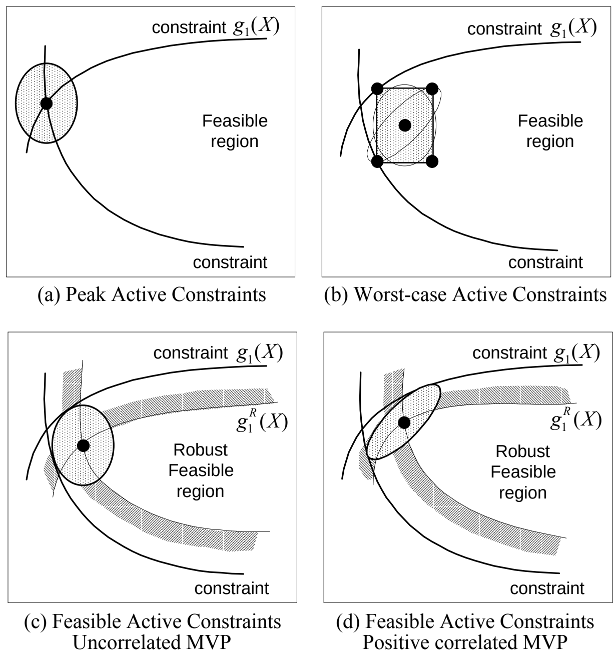



Figure 8. Constraint activity for several variation patterns

Previous studies (Sundaresan *et al.*, 1993; Parkinson *et al.*, 1990) modified the feasible region using a first order Taylor expansion to accommodate the ropagating variation" in constraints:

$$g_j(X, b_j) + \sum_{i=1}^{n} \left| \frac{\partial g_j}{\partial x_i} \Delta x_i \right| \le 0 \quad (15)$$

The modification becomes doubtful if significant nonlinearity and variation correlation are present. In fact, the constraints are deterministic, but the design variables scatter due to manufacturing variations. The overall probability of feasibility is the real concern. Manufacturing Variation Patterns provide a better explanation and quantification of constraint uncertainty. This study formulates the constrained robust optimization as follows:

Minimize $F(X) = \mu_y + \beta^*\sigma_y \approx EP + \beta^*DI$ (16)
Subject to $E[h_k(X)] = 0$ *for* $k=1,2,...,K$ (17)
$\forall$ X $\in$ $MVP(1-\alpha)$, $g_j(X) \le 0, j=1,J$ (18)
where $(1-\alpha)$ is the confident coefficient of the *MVP*

Figures 8(c) & 8(d) illustrate examples of the Feasible Active constraint. The definition of the Feasible Active Constraint using *MVP* is as follows:

Def. 2: Feasible Active Constraint

For $X_{Target}$ with a given *MVP(1-α),* an $i^{th}$ inequality constraint $g_i$ is considered *Feasible Active* at the confidence of *(1-α)*100%* if

1) $\forall X \in MVP(1-\alpha)$ , $g_i(X) \le 0$

2) $\exists X \in MVP(1-\alpha)$ , $g_i(X)=0$

## APPLICATION: MOLDED HELICAL GEARS WITH MINIMUM TRANSMISSION ERROR

### Background

Manufacturing errors and shape deformations of tooth profile due to gear load, when coupled with the shaft misalignment, increase the transmission error drastically. Gear profile modification has been an effective technique to reduce the peak-to-peak transmission error (PPTE) due to manufacturing errors and the elastic deflection of gear teeth under loads (Sundaresan, 1992). Robust optimum seeks the gear designs with the least expected PPTE, while the performance is less sensitive to profile errors, shaft misalignment, and load variations (Yu, 1995).

Welbourn (1979) defined the transmission error as he difference between the actual position of the output gear and the position it would occupy if the gear drive is perfect (infinite stiffness and conjugate teeth)." The modifications enable the unloading of one mating tooth pair when the second pair makes initial contact, which lessens the sudden increase and decrease of mesh stiffness and reduces the variation of transmission error.

Figure 9 illustrates the concept of profile modification. This study uses the Load Distribution Program (1991) to predict the transmission error. LDP characterizes the modification with two parameters: 1) the starting roll angle

$$\varphi = \sqrt{\left(\frac{R_s}{R_b}\right)^2 - 1} \tag{19}$$

and 2) the amount of tip relief $\delta_T$ of the gear profile modification. The modification can be either linear or parabolic.

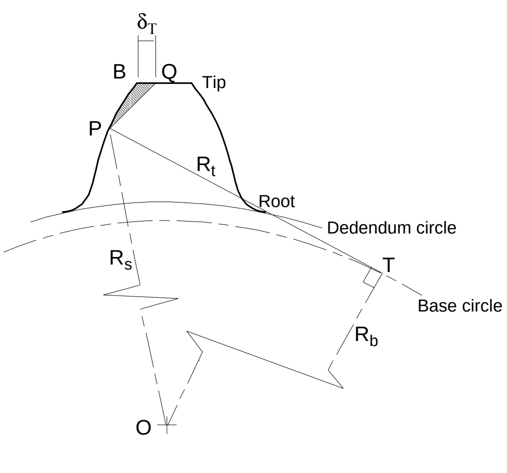



Figure 9. Gear profile modification

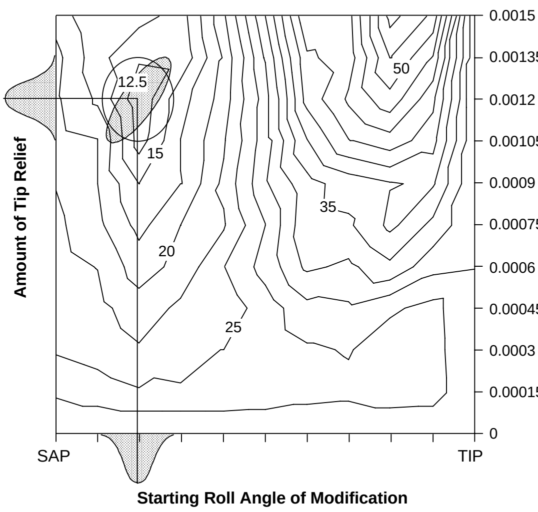



Figure 10. Contour plot of the peak-to-peak transmission errors

Figure 10 shows the contour plot of the peak-to-peak transmission error of the helical gears used in the previous study (Yu, 1995). The lowest contour, 12.5 micro inches, encloses the optimum profile modification that minimizes the PPTE. However, the design is liable to manufacturing and operational variations. The manufacturing errors of the tip relief and the starting roll angle are approximately 0.00015 inch and 1.5 degrees respectively, which induce a significant variation of the performance.

Sundaresan *et al.* (1991) adopted Taguchi orthogonal array and the Sensitivity Index in the optimization process to achieve a statistical optimum. The optimum shows a slight increase of the nominal PPTE but greatly improves the worst case performance, which provides a better overall quality than the conventional peak optimum. However, Sundaresan did not consider interaction effects and possible variation correlation of molded parts such as plastic gears. The observation of Figure 10 suggests that any correlation between the tip relief and the starting roll angle will result in an oblique *MVP* which introduces different performance variations. The optimum assuming independence of variables therefore become doubtful. The application of our proposed design methodology extends the study to the design of molded gears where geometric variables correlate with each other due to material shrinkage.

### The Profile Modification of Molded Gears Using Robust Optimization

The profile modifications of molded gears will be embedded in the mold designs. The distribution of part dimensions is due to the variations of process variables such as mold temperature, packing pressure, and cooling speed. The dimensional variations are coupled because of the material shrinkage, which introduces correlation among geometrical variables. Processes, material properties, and feature characteristics affect the correlation level and the variation patterns.

This study considers the correlation between the tip relief and the starting roll angle, and assumes the correlation coefficient to be 0.7. To reduce the size of experiments, this study also assumes the interaction effects are negligible and the dimensional variations of gears and pinions are independent. The design ranges of the variables are as follows:

1) Starting roll angle of modification on the pinion $\varphi_p$ from the Start radius of Active Profile (SAP) to the tip of the tooth.
2) Starting roll angle of modification on the gear $\varphi_g$ from SAP to the tip of the tooth.
3) Amount of tip relief on the pinion tooth $\delta_{Tp}$ from 0.0 to 0.0015".
4) Amount of tip relief on the gear tooth $\delta_{Tg}$ from 0.0 to 0.0017".
5) Amount of lead modification $\delta_{Lg}$ at both end faces of the pinion tooth from 0.0 to 0.0005". The lead modification was parabolic with zero at the center of the face width. The amount of modification at both end faces of the pinion tooth was assumed to be equal. The gear tooth was unmodified in the lead direction.

The optimization considers four variations:

1) Variation of 0.00015 inch in the parabolic tip relief of pinion and gear
2) Variation of 1.5 degrees in the starting roll angle of pinion and gear
3) Torque ($T$) variation of 200 lb-ins
4) Shaft misalignment $\delta_S$ of 0.0005 inch per inch of face width

The study selects the experimental design of $2^{6-3}$ Fractional Factorial augmented with center point. Unlike full factorial design which needs 64 experiments, this design uses only nine experiments. The design is of resolution III which allows us to investigate all the six main effects if the interactions are negligible. The augmentation of the center point in our experimental design estimates the combining effect of the significant quadratic terms (Yu, 1995). The variations of the PPTE due to the errors of the tip relief and the starting roll angle are significantly nonlinear as shown in the contour plot.

The design adopts the Quadrature Factorial which selects the quadrature points as experimental levels. This study assumes normal variables, and the given variations represent three times of the standard deviations. The corresponding high and low levels are $\mu_i \pm \sqrt{3}\sigma_i$ for independent variables. However, the variations of tip relief and starting roll angle are correlated due to material shrinkage. The quadrature points at the principal axes of the MVP become the experimental setting of the tip relief and the starting roll angle.

Table 1. Comparison of various optimization results

| PARAMETERS | | Peak | Statistical | Robust |
|---|---|---|---|---|
| $\varphi_p$ | (deg.) | 11.45 | 16.57 | 14.89 |
| $\varphi_g$ | (deg.) | 13.01 | 12.81 | 13.19 |
| $\delta_{Tp}$ | ($10^{-3}$ in) | 1.175 | 1.053 | 0.998 |
| $\delta_{Tg}$ | ($10^{-3}$ in) | 1.341 | 1.144 | 1.000 |
| $\delta_L$ | ($10^{-3}$ in) | 0.283 | 0.000 | 0.149 |
| Target PPTE | (μ in) | 5.84 | 10.00 | 16.48 |
| Exp. PPTE (*EP*) | (μ in) | 19.65 | 17.87 | 13.70 |
| Dev. of PPTE (*QI*) | (μ in) | 11.89 | 7.83 | 6.04 |
| Obj. = (*EP*+2**QI*) | (μ in) | 43.44 | 33.53 | 25.77 |

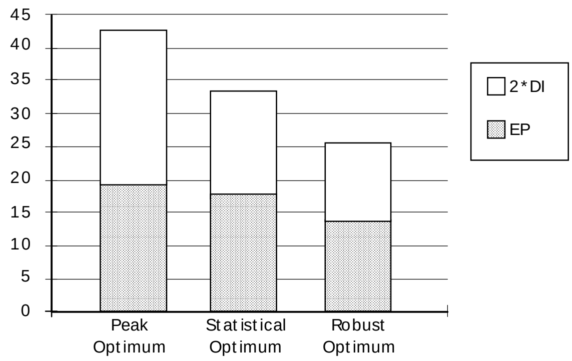



Figure 11. Various optimization results

The Quality Coefficient of Equation (6) is set at 2.0 (Yu, 1995). This study selects the Broydon-Fletcher-Goldfarb-Shanno (BFGS) variable metric method in the search of optimum. Table 1 shows the optimization results. Peak optimization uses the nominal PPTE as design objective and thus has the lowest nominal value comparing with the robust optimum. However, due to the system nonlinearity and the correlation among design variables, the expected PPTE could be quite different from the nominal value. The peak optimum though contains the least nominal PPTE (5.48 μin), the expected PPTE of the design is much higher (19.65 μin). The statistical optimum using Sundaresan procedure improves the design; however, the Robust Optimum presents the lowest expected PPTE and the lowest Deviation Index. Table 1 shows that the Robust Optimum reduces the expected PPTE by 30% and the performance deviation by 49% compared with the Peak Optimum. Figure 11 shows that the statistical worst case of the PPTE could be as high as 43.4 μin for the peak optimum while only 25.8 μin for the robust optimum.

## CONCLUSION

This paper addressed the parameter variations due to interacting manufacturing errors and their impact to the design robustness and constraint activity. The advocacy of the *Manufacturing Variation Pattern* promotes the understanding of the characteristics of manufacturing processes and the effect of the levels of correlation among variables. The variation pattern is crucial to the determination of design robustness. The extension to the robust design concept led to a new definition of constraint activity. The proposed algorithm provides a better measure of design variations than the previous study which assumed independence among the variations of design variables and considered only the worst case within the tolerance space.

The design of molded helical gears with minimum Peak-to-Peak transmission error illustrates our scheme of robust optimization using the Manufacturing Variation Pattern. The application of the concept of *MVP* in conjunction with the Quadrature Factorial techniques better estimates the expected PPTE and the performance deviation in spite of the correlation of variation due to manufacturing processes. The robust optimum not only contains the least expected PPTE but also the minimum sensitivity to the manufacturing variations compared with the results from the previous study.

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## 運用製造變異模態進行最佳設計的穩健化


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摘要


本文研究設計參數因製造方式造成的分佈特性，以及在最佳化過程中對設計穩健性與合理性所造成的影響。作者提出製造變異模態來說明參數的分佈特性與關連性，並分析不同的變異模態所造成的輸出值偏移與設計限制的不確定性。本文結合製造變異模態的觀念與統計實驗計畫於限制最佳化設計的搜尋中，在降低目標函數期望值與對參數變異敏感度的同時，確保設計的合理性。文中探討設計參數因各種典型製造程序所形成的變異模態，並以模造成型件如塑膠與壓鑄齒輪的設計為例，探討因材料收縮造成尺寸參數變異的關連與對傳動輸出損耗分佈的影響，並應用所提出的設計法則，求得成型齒輪的穩健最佳化設計。



Keywords: Robust Design, Factorial Experiment, Taguchi Method, Statistical Optimization, Constrained Optimization, Constrain