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Design for Robustness Based on Manufacturing Variation Patterns

This paper addresses interacting manufacturing errors and their impact on the design robustness and constraint activity. Manufacturing errors often affect design variables with characteristic patterns. This paper defines the Manufacturing Variation Pattern (MVP) to represent this characteristic and investigates its effects. The application of the concept of MVP to design optimization leads to an improved robust optimum. The design of molded gears with minimum transmission error illustrates the proposed scheme's effectiveness. Our model readily accommodates correlation among dimensional errors and significantly reduces performance variation.

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1 Introduction

Recent advances in quality engineering urge designers to consider variations that may be related to material properties, manufacturing process, and operation conditions. Quality products must perform to specifications despite these variations. Taguchi (1978) introduced the concept of *parameter design* which reduces deviation in performance by reducing the sensitivity of the design to variations rather than controlling the sources.Ó d'Entremont & Ragsdell (1988) and Chang (1989) adopted Taguchi's concept of quality-loss to minimize performance variability. Sandgren (1989), Eggert & Mayne (1990), Sundaresan *et al.* (1991), and Yu & Ishii (1993) integrated design performance and variation into the objective function. However, Taguchi's approach to experimental design does not clearly address potential interactions between controllable factors. D'Errico & Zaino (1988) extended Taguchi's approach and advocated a modified approximation using the Gaussian-Hermite quadrature integration. Yu & Ishii (1993) proposed the Fractional Quadrature Factorial to estimate the performance mean and the robustness for applications with significant interaction and nonlinear effects. Design variables are subject to variations that lead to constraint uncertainties. Conventional actively constrained optimum may not be statistically feasible. Parkinson *et al.* (1993) explored the influence of correlated constraints on feasibility and advocated a two-step solution to modify the feasible region. Sundaresan *et al.* (1993) compared the efficiency of three different methods that incorporate variations in constraints. Most of these studies use worst-case analysis and fall short of addressing the variation characteristics.

This paper focuses on the variations of design variables due to interacting errors. Here, the design variables include controllable variables whose values can be selected by designers and uncontrollable parameters whose values are fixed as part of the specifications. Manufacturing errors often affect design variables with characteristic patterns. These "patterns" are particularly important in net shape manufacturing, such as injection molding where the dimensional error is largely due to shrinkage that simultaneously affects multiple variables. One can no longer assume that variations are independent. Few studies have addressed the interdependency among variations on design variables and constraints. This paper introduces the concept of Manufacturing Variation Pattern *(MVP)* to characterize the coupled variations. Different designs and manufacturing processes have unique variation patterns. The performance variation within the pattern will determine the robustness. The paper develops a systematic procedure to 1) identify the variation patterns for typical processes; 2) approximate performance based on the pattern and 3) develop a scheme to find the robust optimum that matches the pattern. The design of molded helical gears with minimum transmission error serves as an illustrative example.

2 Review of Robust Optimization

2.1 Robust Optimization Modeling. The conventional optimization minimizes the nominal performance:

Minimize	$y(X)$	(1)
Subject to	$g_j(X) \le 0$ for j=1,2,...,J	(2)
	$h_k(X) = 0$ for k=1,2,...,K	(3)
	$X=(x_1, x_2,...., x_n)^T$	(4)

The symbol x_i represent design variables, such as geometry, material, and manufacturing parameters. The objective *y(X)* is a function of x_i and will thus have a statistical distribution. The design vector *X* is subject to inequality constraints $g_j(X)$ and equality constraints $h_k(X)$. Probabilistic optimization (Siddall, 1984) takes into account the uncertainties of variables and minimizes the expected mean of the objective.

Minimize
$$\mu_y = E[y(X)] = \int_{X^-}^{X^+} y(X)\cdot p(X)\cdot dX \qquad (5)$$
where *p(X)* is the joint probability function of X

However, this model does not address performance variation. Chen *et al.* (1996) applied the goal programming approach and used the lexicographic minimum principle to model these two objectives at their respective priority levels. Yu & Ishii (1993) adopted the concept of statistical worst case, and suggested the objective consisting of the expected performance and a weighted standard deviation to seek the robust optimum.

$$F(X) = \mu_y + \beta^* \sigma_y \qquad (6)$$
where β is termed the Quality Coefficient

The objective *F(X)* represents the statistical worst response at the confidence level corresponding to β if the distribution of *F(X)* is near normal., i.e., the selection of β controls the balance between target value and variation. Higher weight on *DI* leads to smaller variation, while a lower value shifts the design to the probabilistic optimum.

2.2 Estimations of Expected Mean and Variance. The evaluation of the objective function in robust optimization often requires information of the expected means and the variances of performance. The exact evaluations involve a computationally expensive integration with the joint probability function of control variables. The Taylor expansion approximation leads to the following:

$$\mu_y \approx y(M) + \frac{1}{2}\sum_{i=1}^{n}\left(\left.\frac{\partial^2 y}{\partial x_i^2}\right|_M\right)Var(x_i) + \sum_{j>i}^{n}\sum_{i=1}^{n}\left(\left.\frac{\partial^2 y}{\partial x_i \partial x_j}\right|_M\right)Cov(x_i, x_j) \quad (7)$$

$$\sigma_y^2 \approx E\{[\sum_{i=1}^{n}\left(\left.\frac{\partial y}{\partial x_i}\right|_M\right)(x_i - \mu_i) + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\left.\frac{\partial^2 y}{\partial x_i \partial x_j}\right|_M\right)(x_i - \mu_i)(x_j - \mu_j) - \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\left.\frac{\partial^2 y}{\partial x_i \partial x_j}\right|_M\right)Cov(x_i, x_j)]^2\} \quad (8)$$

where $M = E[(x_1, \ldots, x_n)] = (\mu_1, \mu_2, \ldots, \mu_n)$
$Var(x_i)$ is the variance of x_i.
$Cov(x_i, x_j)$ is the covariance of x_i and x_j

These approximations require the evaluation of second response derivatives. If the evaluation involves complex simulations or experiments, the computational process will be prohibitive. To reduce the evaluation complexity, Yu & Ishii (1993) proposed a zeroth order Fractional Quadrature Factorial Model that combines fractional factorial and Gaussian-Hermite integration. Quadrature Integration provides superior approximations even if the model contains significant interaction and non-linearity effects.

$$\mu_y \approx EP = \sum_{j=1}^{N} W_j \cdot y_j \quad (9)$$

$$\sigma_y^2 \approx DI^2 = \sum_{j=1}^{N} W_j \cdot \left(y_j - E(y)\right)^2 \quad (10)$$

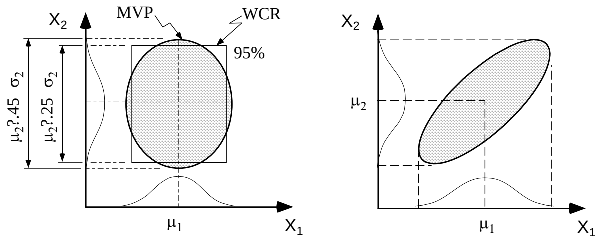
where $N=3^n$
$W_j = w_1 * w_2 \ldots * w_n$
w_i are the weightings of design variable x_i at their respective levels
EP (Expected Performance) is an estimate of expected mean
DI (Deviation Index) is an estimate of the standard deviation of performance

3 Manufacturing Variation Patterns

3.1 Definition. Manufacturing errors often induce scatter to design variables that may be correlated. Most robust design schemes to date use the worst case region (WCR) to represent the variation. Consider two independent normal variables. Conventional worst case regions use the Bonferroni method (Rawlings, 1988). The overall confidence coefficient uses the product of all the univariate confidence coefficients and does not take into account the joint distribution of the variables. Figure 1(a) shows that the intersection of two 97.5% univariate confidence intervals leads to a rectangular region with a simultaneous confidence coefficient of 0.95. It would be misleading to interpret the rectangular intersection as a joint confidence region since some designs inside the pattern are unlikely combinations of the design variable. The variation pattern should be the possible combination of the variables at the specified probability. This paper proposes the concept of the *Manufacturing Variation Pattern* as follows:

Definition 1: Manufacturing Variation Pattern (*MVP*)

Manufacturing Variation Pattern is a set of samples that belong to the (1-α)* 100% joint confidence region of the nominal design. *MVP(1-α)* denotes the space of possible parameter combinations at the confidence coefficient of (1-α) where α indicates the probability of the design distribution outside the variation pattern.



(a) 95% Worst case region and Uncorrelated MVP(0.95)

(b) Positively correlated MVP

Figure 1. Worst case region and Manufacturing Variation Patterns

The distributions of the design variables determine the shape of the *MVP*. The pattern will be a rectangular solid *only* if the distributions of the design variables are uniform. The design variables assume normal distributions in many manufacturing processes, which result in the *MVP* of an orthogonal ellipsoid (Johnson & Wichern, 1988). The confidence coefficient (1-α) determines the size of ellipsoid. Figure 1(a) shows the *MVP(0.95)* of two independent normal variables. The covered areas of WCR and *MVP* are quite different even if the variables are independent. Variation correlation changes the orthogonality of the *MVP*. If the variations are perfectly correlated, one can identify a relationship between the variations of x_i. The corresponding *MVP* becomes a line or a curve. Partial correlation between design variables changes the *MVP* to an oblique ellipsoid. The axes of the ellipsoid show the direction of correlation between design variables. Figure 1(b) shows a typical pattern for a positively correlated *MVP*. Design robustness and feasibility are directly related to the variation pattern of design variables. Engineers should study the correlation among the variables, since erroneous assumptions will lead to inferior designs.

3.2 Manufacturing Variation Patterns for Typical Processes. The orthogonal ellipsoid represents a typical MVP for conventional processes such as lathe-turning, grinding, and milling. The distributions of the part dimensions are often normal and independent. Dimensions *W* and *D* of the lathe-turned shaft in Figure 2 are typical examples. However, many post-machining processes such as heat treatment will change the independence among variables, e.g., dimensions *W* and *D* of the shaft will distort after through hardening. The volume change due to phase transformation lead to dimensional distortions, but interactions of thermal and transformation stresses further complicate the matter. According to Ameen's Rule (1940), dimensional changes resulting from temperature induced stresses will cause the shape of a component to become more spherical, which introduces a negative correlation between *W* and *D*. Figure 2 presents the variation pattern of the heat treated shaft taken from actual production data. The hardening processes consist of a furnace heating and a solution quenching.

The shaft is then tempered to reach the hardness requirement. The *MVP* clearly shows a negative correlation between the width and the spline diameter over pins.

The dimensional changes of molded or die-cast parts have another correlation pattern. Volumetric shrinkage ν affects the dimensional changes of plastic parts. If the material is homogeneous and the cooling and packing variations are negligible, the linear shrinkage rate will be homogeneous and approximately $\nu^{1/3}$. For more complex parts such as a plastic switch housing (Busick, 1994), the variations of packing pressure, mold temperature, melt temperature, and the interaction of geometric features become significant. Figure 3 shows the *MVP* of production parts. Dimension x_1 exhibits primarily linear shrinkage, and dimension x_2 exhibits both linear shrinkage and warpage. The oblique pattern shows a strong positive correlation between x_1 and x_2.

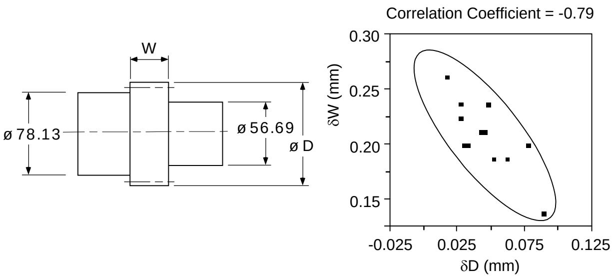


Figure 2. Heat treated spline shaft and the *MVP*(0.95) from the experimental data

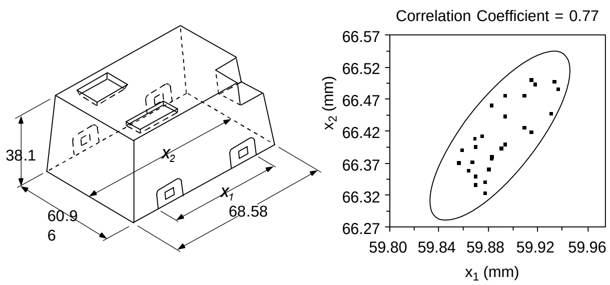


Figure 3. Plastic switch housing and the *MVP* (0.95) from experimental data

3.3 Characterizing the Manufacturing Variation Patterns. This paper applies the multivariate statistical techniques (Johnson & Wichern, 1988) to characterize the variation patterns. Typical mass production processes usually have well-established statistical information of design variable. Consider normal variables x_i with means μ_i and variance-covariance σ_{ij}, the formulation for the n dimensional ellipsoid of the *MVP(1-*α*)* is as follows:

$$(X-M)^T\Sigma^{-1}(X-M) \le \chi^2_{(n,\alpha)} \tag{11}$$

where $X = [x_1,....,x_n]^T$ including controllable variables and uncontrollable parameters

$M = E[x_1,....,x_n]^T = [\mu_1, \mu_2,..., \mu_n]^T$

Σ is the variance-covariance matrix of X

$\chi^2_{(n,\alpha)}$ is the value of the chi-square value with n degrees of freedom that leaves probability α in the upper tail.

The axes of the ellipsoid lie in the directions of the eigenvectors, e_i, of Σ. The lengths of the principal axes are equal to $\sqrt{\chi^2_{(n,\alpha)}\lambda}$, where λ_i are the eigenvalues of Σ. If the variables x_i are independent, the formulation can be simplified as follows:

$$\sum_{i=1}^{n}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^2 \le \chi^2_{(n,\alpha)} \tag{12}$$

The formulation corresponds to an orthogonal ellipsoid with the lengths of the principal axes $\sqrt{\chi^2_{(n,\alpha)}}\sigma_i$. In practice, we use the sampling average $\bar{x}_i$ to estimate μ_i, and the sampling standard deviation s_i to estimate σ_i. The correlation coefficient r_{ik} measures the strength of the linear association between two variables, x_i and x_k.

$$r_{ik} = \frac{s_{ik}}{\sqrt{s_i^2}\sqrt{s_k^2}} = \frac{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)(x_{kj}-\bar{x}_k)}{\sqrt{\sum_{j=1}^{n}(x_{ij}-\bar{x}_i)^2}\sqrt{\sum_{j=1}^{n}(x_{kj}-\bar{x}_k)^2}} \tag{13}$$

The variables assume independence if the corresponding correlation coefficient r is not significantly different from zero. The sign of r indicates the direction of the association: positive r implies a tendency for both values to be larger or smaller than their average values together, and negative r implies a tendency for one value to be larger than its average when the other is smaller than its average. If r is close to 1 or -1, a functional relation may exist between these two variables. Figure 4 shows the *MVP*(0.50) and the *MVP*(0.95) of a bivariate normal example with common variance. For variables with other distribution patterns or nonlinear correlation, their *MVP*s will be much more complicated. However, the *central limit* effect of statistics suggests that the sampling distributions of many multivariate statistics are approximately normal, regardless of the form of the parent population.

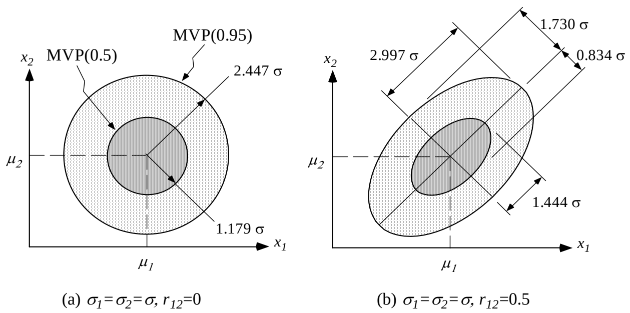


Figure 4. *MVP*(0.50) and *MVP*(0.95) of a bivariate normal example

4 MVP and Constrained Robust Optimization

4.1 Design Robustness. The proposed method combines the concept of *MVP* and the Quadrature Factorial Model (Yu & Ishii, 1993) to evaluate the objective function. For independent normal variables, we can directly apply equations (9) and (10) to evaluate *EP* and *DI*. For the *MVP* of correlated variables, a decoupling transformation of the variables is required to identify the quadrature factorial. The transformation between these coordinates is:

$$X = M + [e_1, e_2, \cdots, e_n]Z \tag{14}$$

where $X = [x_1,....,x_n]^T$

$Z = [z_1,\ldots,z_n]^T$
$M = [\mu_1, \mu_2,\ldots, \mu_n]^T$
e_i are the eigenvectors of the variance-covariance matrix Σ

The factorial experiments should select the quadrature points along the Z axes (Figure 5). The corresponding high and low levels in terms of Z coordinates are $\pm\sqrt{3\lambda_i}$, where λ_i are the eigenvalues of Σ.

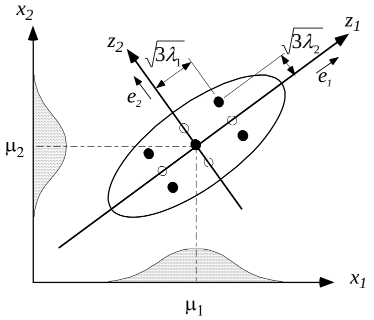


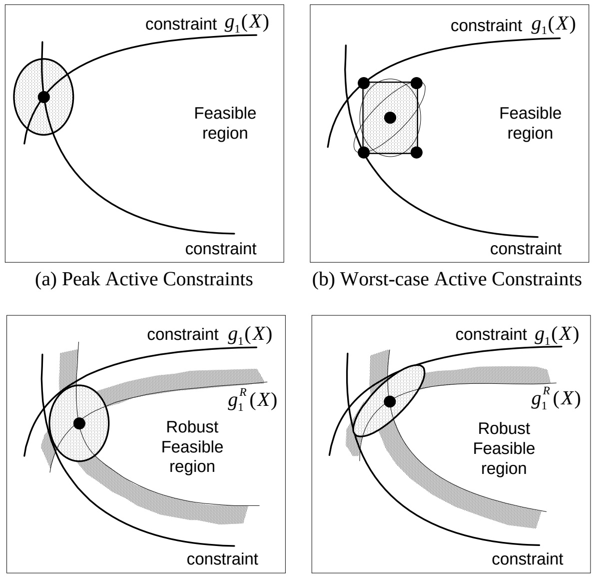
Figure 5 Transformation of factorial experiments to decouple correlated *MVP*

4.2 Constraint Uncertainty. Manufacturing errors introduce variations to design variables that propagate to constraints. Robust optimization uses statistic techniques to extend the definitions of constraints. A constrained optimum should be statistically feasible regardless of constraint uncertainties. This paper defines the robust feasibility as no constraint violation within the *MVP*. Conventional peak constrained optimum may contain a large portion of unsatisfactory designs (Figure 6a). One resolution is to move the design to the worst-case actively constrained point as shown in Figure 6(b), but the worst-case region does not capture the actual distribution of design variables. The constrained solution may not be the true optimum as we examine the overlay *MVP*s in Figure 6(b). The worst-case actively constrained design may be over or under constrained depending on the actual variation patterns. Parkinson *et al.* (1993) suggested a modification of the feasible region using the first order Taylor's approximation to accommodate the "propagating variation" in the constraints:

$$g_i(X) + \sum_{j=1}^{n} \left| \frac{\partial g_i}{\partial x_j} \Delta x_j \right| \le 0 \qquad (15)$$

The modification becomes uncertain if significant non-linearity and variation correlation are present. The constraints are deterministic, but the design variables scatter due to manufacturing variations. The overall probability of feasibility is the real concern. *Manufacturing Variation Pattern* provides a better quantification of constraint uncertainty. This concept formulates the constrained robust optimization as follows:

Minimize	$F(X) = \mu_y + \beta^*\sigma_y \approx EP + \beta^*DI$	(16)
Subject to	$E[h_k(X)] = 0$ for $k=1,2,\ldots,K$	(17)
	$\forall$ X $\in$ *MVP*(1-α), $g_j(X) \le 0$, $j=1,J$	(18)
where	(1-α) is the confident coefficient of the *MVP*	



(a) Peak Active Constraints (b) Worst-case Active Constraints

(c) Feasible Active Constraints Uncorrelated MVP (d) Feasible Active Constraints Positive correlated MVP

Figure 6. Constraint activity for several variation patterns

Figures 6(c) & 6(d) illustrate examples of the Feasible Active constraint. An inequality constraint is considered *Feasible Active* if the *MVP(1-α)* of the design is inside the feasible region and tangent to the original constraint surface $g_i(X)$. The design variables X include controllable variables and uncontrollable parameters. Controllable variables vary in the design ranges and uncontrollable parameters remain fixed in the search of optimum. The definition of the Feasible Active Constraint is as follows:

Definition 2: Feasible Active Constraint

For design X_0 with a given *MVP(1-α)*, inequality constraint $g_i(X)$ is considered *Feasible Active* at the confidence of *(1-α)*100%* if

1) $\forall X \in MVP(1\text{-}\alpha)$, $g_i(X) \le 0$
2) $\exists X \in MVP(1\text{-}\alpha)$, $g_i(X)=0$

The locus of the centroid of the *MVP* tangent to the original inequality constraint $g_i(X)$ composes the *Robust Inequality Constraint* $g_i^R(X)$. Confining the designs inside the region bound with $g_i^R(X)$ in the search process will ensure that the variation patterns are inside the feasible region of the original constraints. However, the details of the application procedure still await future investigation.

5 Application: Molded Helical Gears

5.1 Background. Manufacturing errors and shape deformations of gear tooth under load coupled with shaft misalignment increase the transmission error drastically. Gear profile modification has been an effective technique to reduce the peak-to-peak transmission error (PPTE). Robust optimization seeks the gear designs with the least expected PPTE, where the performance is less sensitive to these variations. Welbourn (1979) defined the transmission error as "the difference between the actual position of the output gear and the position it would occupy if the gear drive is perfect (infinite stiffness and conjugate teeth).Ó Profile modification enables the unloading of one mating tooth pair when the second pair makes initial contact, which lessens the sudden increase and decrease of mesh stiffness and reduces the variation of transmission error. Figure 7 illustrates the concept of profile modification. This study uses the Load Distribution Program (Houser, 1991) to predict the transmission error. LDP characterizes the profile modification with two variables: 1) the starting roll angle (eq. 19) and 2) the amount of tip relief δ_T. The modification can be either linear or parabolic.

$$\varphi = \sqrt{\left(\frac{R_s}{R_b}\right)^2 - 1} \qquad (19)$$

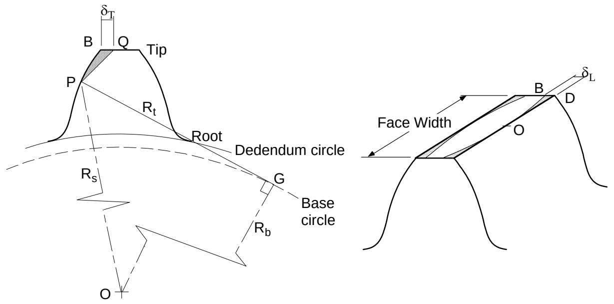


Figure 7. Gear profile modification

Figure 8 shows the contour plot of the PPTE of the helical gears in Table 1. The plot assumes the parabolic modifications with equal amounts of tip relief on the gear and the pinion and a fixed 7.6μm parabolic lead modification. The starting roll angle of modification on the gear is varied proportionately with the pinion. The design is susceptible to manufacturing and operational variations. The manufacturing errors of the tip relief and the starting roll angle are approximately 3.8μm and 1.5 degrees respectively. Sundaresan *et al.* (1991) adopted Taguchi's orthogonal array and the Sensitivity Index in the optimization process to achieve a statistical optimum. The statistical optimum shows a slight increase of the nominal PPTE but greatly improves the worst case performance, but Sundaresan did not consider interaction effects and possible variation correlation. The observation on Figure 8 suggests that any correlation between the tip relief and the starting roll angle will result in an oblique *MVP* that introduces different performance variations. The application of our proposed design method extends the previous study to the design of molded gears where geometric variables correlate with each other due to material shrinkage.

Table 1. Helical Gear Geometry

GEOMETRIC PARAMETERS	Pinion	Gear
Transmitted torque (N-m)	84.76	
Center Distance (mm)	71.0	
Normal diametral pitch (1/mm)	0.474	
Normal pressure angle (degree)	16.0	
Helix angle (degree)	30.0	
Profile contact ratio	2.02	
Face contact ratio	1.13	
Total contact ratio	3.15	
Number of teeth	18	41
Face width (mm)	15.24	15.24
Outer diameter (mm)	52.484	101.745
Root diameter (mm)	39.413	88.674
Roll angle @ pitch circle	16.70°	16.70°
Roll angle @ outer circle	44.09°	22.44°
Roll angle @ SAP*	3.63°	4.68°

*SAP: The Start Radius of the Active Profile

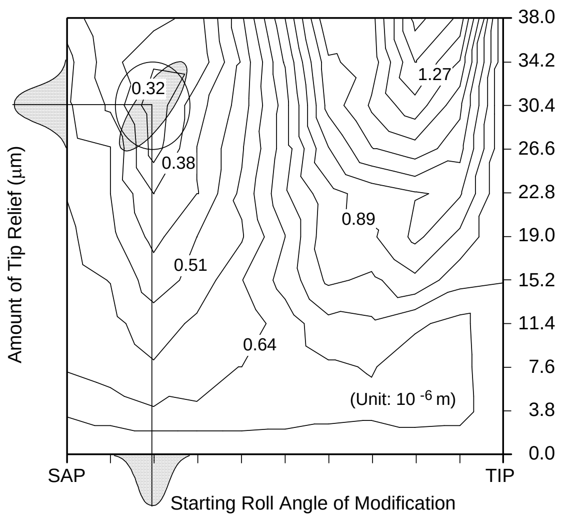


Figure 8. Sample contour plot of the peak-to-peak transmission errors

5.2 The Profile Modification of Molded Gears Using Robust Optimization. The profile modifications will be embedded in the mold designs. The distribution of part dimensions is due to the variations of process parameters such as mold temperature, packing pressure, and cooling time, and the dimensional variations are coupled because of the material shrinkage. Processes, material properties, and feature characteristics affect the correlation level and the variation patterns. Based on industrial practice, this study assumes the correlation coefficient between the tip relief and the starting roll angle to be 0.7. The design ranges of the controllable variables are as follows:

1) Starting roll angle of modification on the pinion (φ_p) from the Start radius of Active Profile (SAP) to the tip of the tooth.
2) Starting roll angle of modification on the gear (φ_g) from SAP to the tip of the tooth.
3) Amount of tip relief on the pinion tooth (δ_{Tp}) from 0 to 38 (μm).
4) Amount of tip relief on the gear tooth (δ_{Tg}) from 0 to 43 (μm).
5) Amount of lead modification (δ_{Lp}) at both end faces of the pinion tooth from 0 to 13 (μm). The lead modification was parabolic with zero at the center of the face width. The amount of modification at both end faces of the pinion tooth was assumed to be equal. The gear tooth was unmodified in the lead direction.

The optimization considers four variations of controllable variables and two variations of uncontrollable parameters:

1) Variation of 3.8 (μm) in both the parabolic tip relieves of pinion and gear
2) Variation of 1.5 (degree) in both the starting roll angles of pinion and gear
3) Torque (T) variation of 22.6 (N-m)
4) Shaft misalignment (δ_S) of 0.0005 mm per mm of face width

The experimental design adopted the 2^{6-3} Fractional Factorial augmented with center point (Montgomery, 1991). Unlike full factorial design which needs 64 experiments, this design uses only nine experiments. Table 2 presents the experimental settings of the design variables to apprehend the output variations. The variations of the PPTE due to the errors of the tip relief and the starting roll angle are nonlinear as shown in the contour plot. The augmentation of the center point provides an estimate of the combining effect of the significant quadratic terms. We also used the Quadrature Factorial that selects the quadrature points as the experimental levels. The design variables assume normal distributions, and the given variations represent three times of the standard deviations of the variables. The variations of the parameters, transmitted torque and shaft misalignment, are independent. The corresponding quadrature levels of the parameters are simply $\mu_i \pm \sqrt{3}\sigma_i$ (Yu and Ishii, 1994), but the variations of tip relief and starting roll angle of the molded parts are correlated due to material shrinkage. The experimental setting of the tip relief and the starting roll angle should select the quadrature points along the principal axes of the *MVP*. The formulation of the *MVP(0.95)* is shown in Equations (20). Consider the section of the variation pattern on the plane of tip relief and starting roll angle of the gear for the simplification of illustration. The application of the procedures illustrated in section 3.3 and 4.1 gives the oblique *MVP*(0.95) and the corresponding factorial designs as shown in Equation (21) and Figure 9.

Table 2. Fractional Factorial Array of the molded gear design

$2^{6\text{-}3}$+1	φ_p	δ_{Tp}	φ_g	δ_{Tg}	T	δ_s
R1	+	-	-	-	-	+
R2	-	+	-	-	+	-
R3	-	-	+	+	-	-
R4	+	+	+	+	+	+
R5	+	+	-	+	-	-
R6	+	-	+	-	+	-
R7	-	+	+	-	-	+
R8	-	-	-	+	+	+
R9	0	0	0	0	0	0

Note: Ò+Ó the high level, Ò-Ó the low level
Ò0Ó the center level.

$$\begin{bmatrix} \varphi_p-\varphi_{p0} \\ \delta_{Tp}-\delta_{Tp0} \\ \varphi_g-\varphi_{g0} \\ \delta_{Tg}-\delta_{Tg0} \\ T-T_0 \\ \delta_s-\delta_{s0} \end{bmatrix}^T \begin{bmatrix} \sigma_{\varphi p}^2 & r\sigma_{\varphi p}\sigma_{Tp} & 0 & 0 & 0 & 0 \\ r\sigma_{\varphi p}\sigma_p & \sigma_{Tp}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\varphi g}^2 & r\sigma_{\varphi g}\sigma_{Tg} & 0 & 0 \\ 0 & 0 & r\sigma_{\varphi g}\sigma_{Tg} & \sigma_{Tg}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_T^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_s^2 \end{bmatrix}^{-1} \begin{bmatrix} \varphi_p-\varphi_{p0} \\ \delta_{Tp}-\delta_{Tp0} \\ \varphi_g-\varphi_{g0} \\ \delta_{Tg}-\delta_{Tg0} \\ T-T_0 \\ \delta_s-\delta_{s0} \end{bmatrix} = \chi^2_{(6,\,0.05)} \quad (20)$$

$$\begin{bmatrix} \varphi_g-\varphi_{g0} \\ \delta_{Tg}-\delta_{Tg0} \end{bmatrix}^T \begin{bmatrix} 0.25 & 0.443 \\ 0.443 & 1.604 \end{bmatrix}^{-1} \begin{bmatrix} \varphi_g-\varphi_{g0} \\ \delta_{Tg}-\delta_{Tg0} \end{bmatrix} = 12.6 \quad (21)$$

The optimization process applies the quadrature points of the *MVP(0.95)* to evaluate the objective function of Equation (16). The Quality Coefficient β is set at 2.0 and the objective will stand for the worst PPTE at the confidence of 97.7%. The Broydon-Fletcher-Goldfarb-Shanno (BFGS) variable metric method reached the optimum result shown in Table 3. Peak optimization uses the nominal PPTE as design objective and thus has the lowest nominal value compared to the robust optimum. Due to the non-linearity of model and the correlation among design variables, the expected PPTE could be different from the nominal value. The nominal PPTE of the peak optimum is 0.148 (μm), whereas the mean PPTE of the design is 0.499 (μm) and the statistical worst PPTE of the design could be as high as 1.103 (μm). The Robust Optimum outperforms the statistical optimum from Sundaresan's procedure; the Robust Optimum reduces the expected PPTE by 30% and the performance variation by 49% compared with the Peak Optimum.

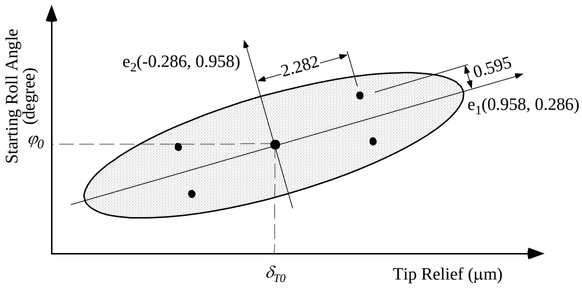


Figure 9. *MVP*(0.95) and the quadrature points of the molded gears/pinions

Table 3 Comparison of the Optimization Results

Design Variables		Peak	Statistical	Robust
φ_p	(deg.)	11.45	16.57	14.89
φ_g	(deg.)	13.01	12.81	13.19
δ_{Tp}	(μm)	29.8	26.7	25.3
δ_{Tg}	(μm)	34.1	29.1	25.4
δ_{Lp}	(μm)	7.2	0.0	3.8
Nominal PPTE	(μm)	0.148	0.254	0.419
Expected PPTE (*EP*)	(μm)	0.499	0.454	0.348
Std.Dev. of PPTE (*DI*)	(μm)	0.302	0.199	0.153
Objective=(*EP*+2**DI*)	(μm)	1.103	0.852	0.655

6 Conclusion

This paper addressed the variations of design variables due to interacting manufacturing errors. The advocacy of the *Manufacturing Variation Pattern* promotes the understanding of the characteristics of manufacturing variation and their effect on robust design and constraint activity. Robust optimization based on *MVP* assures the optimality of performance robustness and design feasibility especially for applications with correlated variations of design variables. The method requires substantial amount of statistical information thus must be obtained through well thought out design of experiments.

The design of molded helical gears with minimum transmission error illustrated the proposed scheme of robust optimization. The application of the concept of *MVP* and the Quadrature Factorial techniques better estimates the expected PPTE and the performance variation despite the correlation among dimensional errors due to manufacturing processes. The robust optimum not only contains the least expected PPTE but also the minimum sensitivity to the manufacturing variations.

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