Residual Stress on Toughening with Spherical Inclusions Accompanying Phase Transformation

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ABSTRACT

A micro-mechanical theory based on the mean-field approach is employed to determine the residual stress due to phase transformation. The stresses of the matrix within transformed zone are evaluated. The derived hydrostatic residual stress is available for the heterogeneous solids, especially, at $c_1 > 0.3$ compared with that evaluated from the continuum theory. The modification factors of the residual stress for composites with different elastic moduli have also found. The results show that the volumetric expansion and softer inclusions can provide more effective toughening. The explicit form of the toughness change is presented for the composite with spherical particles.

INTRODUCTION

In the development of transformation toughening in ceramics such as partially stabilized zirconia (PSZ) and zicornia toughened alumina (ZTA), most people concentrated on the residual strain which induces a compressive traction on the surface of the pre-existing flaw to restrict the crack propagation [1-3]. Alternatively, the so-called *R*-curve behavior can also lead to a maximum in the strength-toughness relation. The measured stress intensity factor, denoted by K_m, rises with increasing crack length for small cracks. The approach to predict the strength-toughness relationships for transformation has been proposed in PSZ materials [4], where the constituents have the same material properties. However, in ZTA materials, for example, the Al_2O_3 matrix and *tetragonal*-ZrO_2 inclusions possess different elastic moduli, and the influence of interaction between two phases becomes important. The intent of the present work is to extend the formulation of the previous model [4], but with considering the inhomogeneity effect based on the Eshelby-Mori-Tanaka theory [5-6].

STRESS RELIEF

It is assumed that the half-height of the transformation zone *H* is small compared with the length of a semi-infinite crack, and the crack propagates along the 1-axis and perpendicular to the 2-axis. For sub-critically transforming materials under steady-state growth conditions [7], the mean residual stress σ_m generated by the dilatant transformation strain ε_{kk}^{ph} in the plane- strain case is given by

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$$\sigma_m = \frac{2}{9}\frac{E}{1-\nu} c_1 \varepsilon_{kk}^{ph} \tag{1}$$

E, ν and c_1 are referred to as the Young Modulus and Poisson ratio of the composite, and the volume fraction of the inclusions, respectively. However, the strengths of the material calculated from the residual stresses in Eq. (1) seemly can not have good agreements with the experimental data in the ductile material and/or the heterogeneous solids [4]. The influence of inhomogeneity in Eq. (1) is neglect as the variations of the elastic moduli in composite and c_1 increase. A general consideration to evaluate the residual stresses in the composite is proposed, now, as follows.

Following the mean-field approach involving inhomogeneity and transformation problems [5-6], the E-M-T theory provides the information about the mean perturbed stress of the matrix in terms of the material properties and the volume concentration. A simple scheme to evaluate the inner product and orientational average of an isotropic tensor, for example, can be found in Pan and Weng [8] for the martensitic transformation and thermal expansion problems. For the spherical inclusions accompanying phase transformation, the hydrostatic residual stress of the matrix $\tilde{\sigma}_{kk}$, after some derivations [5-6,8] for 3-D random orientation of the inclusions, is determined, and the result is

$$\tilde{\sigma}_{kk} = \frac{12 c_1 \kappa_0 \kappa_1 \mu_0 \varepsilon_{kk}^{ph}}{3\kappa_0\kappa_1 + 4\mu_0(c_0\kappa_0 + c_1\kappa_1)} \tag{2}$$

where the subscript character 0 and 1 are referred to as the matrix and the inclusion phase, respectively. The bulk and shear modulus is denoted by κ and μ in turns. The volume fraction of the matrix c_0 is equal to $1-c_1$. This average perturbed stress of the matrix due to the volumetric expansion of transforming inclusions is tensile, and has the relation of $\tilde{\sigma}_{11} = \tilde{\sigma}_{22} = \tilde{\sigma}_{33}$. Assume that the main-crack propagates within the matrix, the stress $\tilde{\sigma}_{22}$ (tensile in the matrix) will create a compressive traction on the crack surface in the wake, so as to toughen the composite. The magnitude of the residual stress $\tilde{\sigma}_{22}$ is

$$\tilde{\sigma}_{22} = \frac{4 c_1 \kappa_0 \kappa_1 \mu_0 \varepsilon_{kk}^{ph}}{3\kappa_0\kappa_1 + 4\mu_0(c_0\kappa_0 + c_1\kappa_1)} \tag{3}$$

The values of the residual stress, determined from Eq. (1) and Eq. (3), are identical while the elastic moduli of both phases in the composite are the same. In other words, the hydrostatic residual stress, in this special case, derived from two different methods— continuum theory and eigenstrain approach-- is exactly equal. The results for the latter theory can show the inhomogeneous effects, whereas the former one does not.

The stress field from the crack in general is

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta), \quad (r \to \infty) \tag{4}$$

where K is referred to as the applied stress intensity factor. The distance r is measured from the crack-tip, and the angle θ is the position counterclockwise from the crack plane. The functions $f_{ij}(\theta)$ are universal and given in many texts on fracture mechanics. Under the small scale transformation condition, the reduction from K to K_{tip} is expected due to transformation, and K_{tip} governs the fracture process at the tip. The stress intensity factor near the crack-tip $K_{\text{tip}} = K + \Delta K$, where ΔK is the toughness change. From the idea of the comparison material [5-6], the stress of the matrix $\sigma_{ij}^{(0)}$ after the transformation, then, can be evaluated by $\sigma_{ij}^{(0)} = \sigma_{ij} + \tilde{\sigma}_{ij}$. Under the Mode I loading and the plane-strain condition, the hydrostatic stress of the matrix near the vicinity of the crack, after phase transformation, can be recast by

$$\sigma_{kk}^{(0)} = \frac{K + \Delta K}{\sqrt{2\pi r}}[2(1+\nu)\cos(\frac{\theta}{2})] \tag{5}$$

where ν is the Poisson ratio of the material within the transformed zone.

TOUGHNESS CHANGE

From dimensional considerations, different from other models [1-3], the steady-state toughness K_m can be expressed by $K_m = K_0 + |\Delta K|$, and the toughness change ΔK due to the residual stress can be expressed by

$$\Delta K = -\beta \tilde{\sigma}_{kk}\sqrt{H} \tag{6}$$

The negative sign of ΔK means the toughening. The parameter β depends on the zone shape or the transformation criterion. It is easy to determine the β value for different materials from the experiments by plotting fracture toughness (K_m) vs transformation zone size ($c_1\sqrt{H}$) [9]. It was found that, for example, β is 0.68 for *Mg*-PSZ and 0.33 for *Y*-TZP (Al_2O_3) materials separately [4]. It is noted that the form of the toughness increment in Eq. (6) due to the residual stress is slightly different from Swain and Rose [4] by using $\tilde{\sigma}_{kk}$ instead of $3\sigma_m/2$. In fact, as one knows, the toughness change in the vicinity of the crack is not always of the negative sign. Zone shape effects on ΔK are very important. For instant, the profile due to dilatant transformation ahead of the crack, precisely in the $-\pi/3 \le \theta \le \pi/3$ region, provides deleterious *positive* ΔK. Thereby, inclusions in front of the crack rise K_{tip}, whereas those at the side $\theta > \pi/3$ reduce the stress intensity factor.

The explicit form of toughness change for materials with spherical inclusions can be reduced to be of the simple form in the following.

$$\Delta K = -\beta \frac{12 c_1 \kappa_0 \kappa_1 \mu_0 \varepsilon_{kk}^{ph}\sqrt{H}}{3\kappa_0\kappa_1 + 4\mu_0(c_0\kappa_0 + c_1\kappa_1)} \tag{7}$$

The toughness change in Eq. (7) can reveal the influence of inhomogenity and volume concentration in the composite.

DISCUSSION

It is now of interest to examine the real material systems. Since the lack of the information about zirconia-toughened concrete, the PSZ and ZTA materials are typically selected to represent the two-phase composite for L_1=L_0 and $L_1 \neq L_0$ case, respectively. L_1 and L_0 are the elastic moduli tensor of the inclusion and the matrix, respectively. The elastic properties of those materials are shown as follows [7, 10-13]:

PSZ materials: the *cubic*-ZrO_2 matrix and *t*- ZrO_2 inclusion with the same elastic moduli.

$$\kappa = 172.5GPa\ ,\quad \mu = 79.6GPa\ ,\quad \varepsilon_{kk}^{ph} = 0.056.$$

ZTA materials: the Al_2O_3 matrix and t- ZrO_2 inclusion.

$$\kappa_0 = 260GPa\ ,\quad \mu_0 = 156GPa\ ,\quad \varepsilon_{kk}^{ph} = 0.047.$$

The toughness for c- ZrO_2, t- ZrO_2 and Al_2O_3 is 3.7, 6.62 and 4.89 $MPa\sqrt{m}$, in turns.

The residual stresses induced by phase transformation are found in Eq. (1) and Eq. (3). Those perturbed stresses are equal to each other in PSZ materials. However, the normalized stresses, $\tilde{\sigma}_{22}/\sigma_m$, depicted in Fig. 1 are different in ZTA materials with change c_1 because of the influence of inhomoheneity, where the effective moduli in σ_m is calculated from the Rule-of-Mixture. The derived residual stress $\tilde{\sigma}_{22}$ is higher than σ_m if $c_1 > 0.3$. This result interprets that σ_m is suitable for the low volume fraction of the inclusions in ZTA materials [4], and the mean residual stress $\tilde{\sigma}_{22}$ might be useful for the middle-high volume concentration. This conclusion is also shown and depicted in Fig. 2 while the influence of inhomogeneity has been investigated. The normalized residual stress vs log(μ_1/μ_0) is plotted in Fig. 2 with $\nu_0 = \nu_1 = 0.3$ and $\mu_1 = 79.6GPa$. The normalized stresses change as the ratios of μ_1/μ_0 increase or decrease. It is necessary to multiply this normalized factor for the results calculated from Swain and Rose's approach [4] if the elastic moduli of the constituents are different.

From Eq. (6), the toughness change is proportional to the residual stress and the square root of the zone size. The volume fraction effect on toughening for all materials is shown in Fig.3 by using $|\Delta K|\ /\beta\sqrt{H}$ vs log(μ_1/μ_0) with $\nu_0 = 0.25$ and $\nu_1 = 0.3$. Obviously, the larger c_1 transforms, the more toughened composite occurs. Besides, the results in Fig. 3 also reveal that the softer inclusions with the harder matrix have more effective transformation toughening.

The stress of the matrix relieved or strengthened after phase transformation depends on the zone shape and the position near the crack. The contour of the transformed zone due to dilatant transformation has the relation of $H = r\sin(\pi/3)$ for $\theta \geq \pi/3$. For Mg-PSZ with $\beta = 0.68$ and $H = 0.6\mu m$ [7], the hydrostatic stresses of the matrix calculated from Eq. (5) and Eq. (7) ahead of the crack at $\theta = 0$ are plotted in Fig. 4 with c_1 from 0 to 0.3. The line with $c_1 = 0$ means no transformation. In Fig. 4, the volumetric expansion of inclusions will increase the hydrostatic stress of the matrix. This explains why many microcracks accompanying with phase transformation take place in front of the crack [9]. The stress of the matrix at $\theta = \pi/3$ and $\theta = \pi/2$ are also displayed in Fig. 5 and Fig. 6, respectively. The stresses within the transformed region at $\theta \geq \pi/3$ are relieved due to the phase transformation, so as to yield the toughening.

The normalized toughness increase is depicted in Fig.7 for PSZ and ZTA materials, where the original toughness K is equal to $c_0K_0+c_1K_1$ and $H = 1.0\,\mu m$ for ZTA [12]. For PSZ, the $\Delta K/K$ is 0.384 at $c_1=0.3$, and it is within the 0.2~0.6 range [1,7]. This means that the theoretic predictions of transformation toughening are reliable. The ratio of toughness vs the volume concentration in Fig. 7 is not of the linear relation, especially, with increasing of c_1.

CONCLUDING REMARKS

One analyzed the mean residual stress induced by the phase transformation in the composite containing the spherical inclusions. This approach allows us to deal with the inhomogeneity effect of the heterogeneous solids. The perturbed stresses calculated from Swain and Rose’s method need to multiply the modification factor, which has been determined here. A simple and reliable method based on the dimensional analysis is proven to determine the toughness change if the parameter β is known experimentally. Actually, this method can also cover all crack-shielding mechanisms due to phase transformation. However, to work well on fracture toughness, the information of parameter β for specific materials has to be established first.

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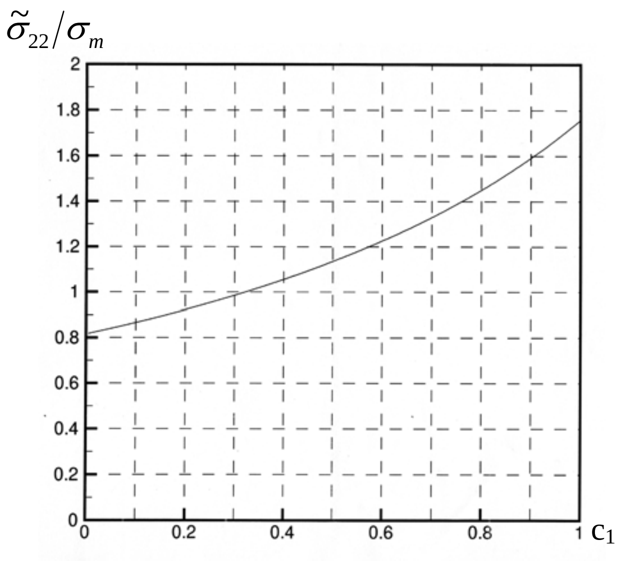

FIG.1 The normalized residual stress and volume concentration in ZTA materials.

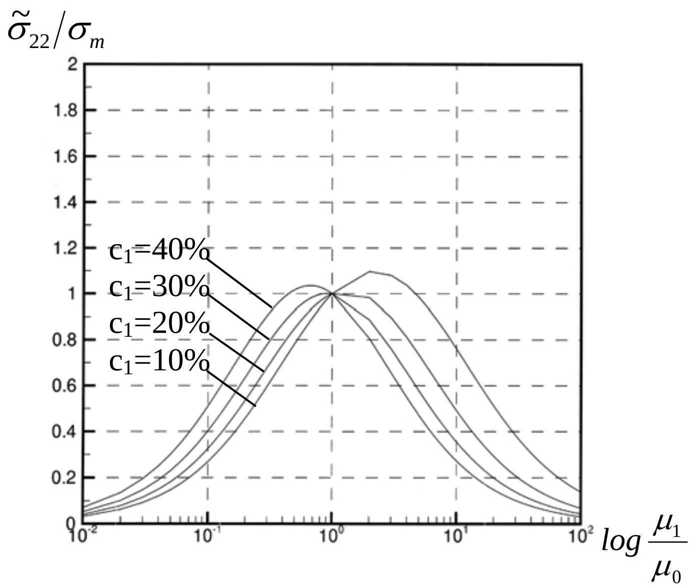

FIG. 2 The normalized residual stress and the ratio of elastic shear modulus at $\nu_0 = \nu_1 = 0.3$.

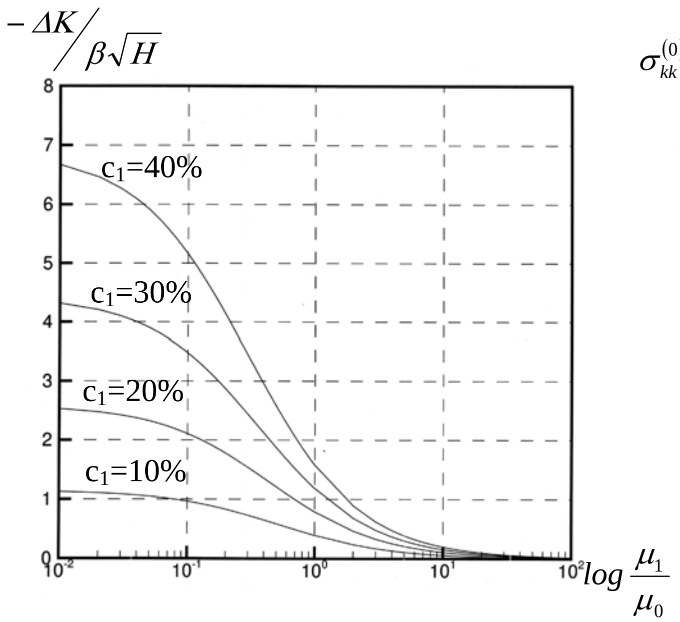

FIG. 3 The toughness increment and the ratio of elastic shear modulus at $\nu_0 = 0.25$ and $\nu_1 = 0.3$.

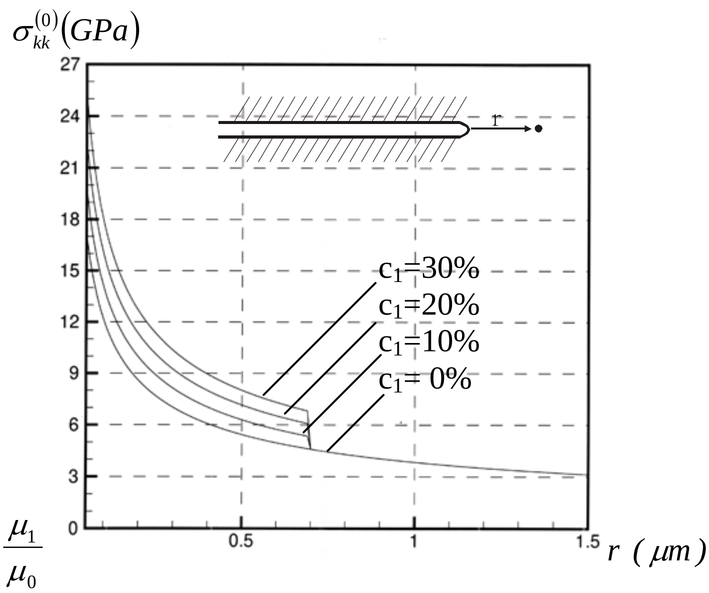

FIG. 4 The hydrostatic stress of the matrix with the distance r from the crack-tip at $\theta = 0$.

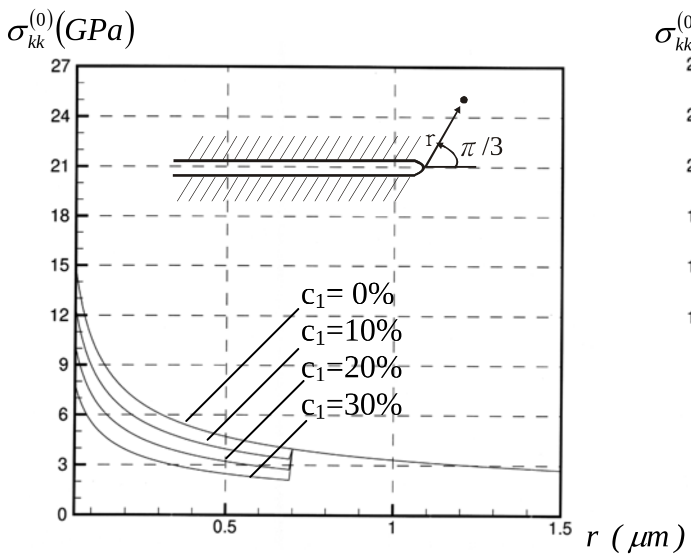

FIG. 5 The hydrostatic stress of the matrix with the distance r from the crack-tip at $\theta = \pi / 3$.

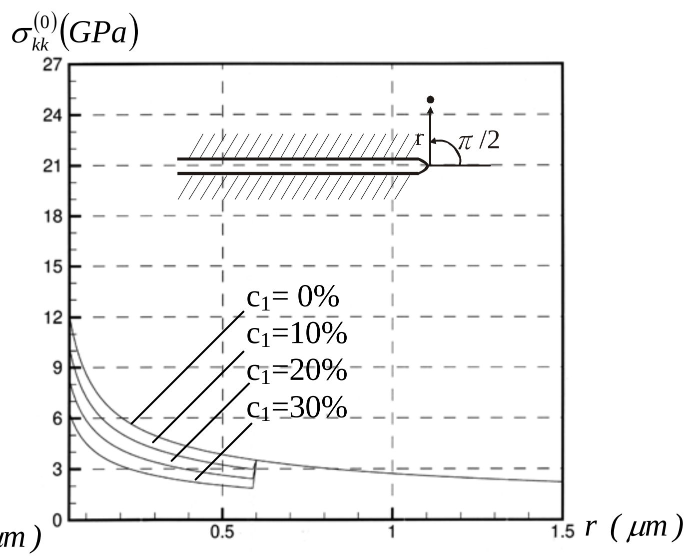

FIG. 6 The hydrostatic stress of the matrix with the distance r from the crack-tip at $\theta = \pi / 2$.

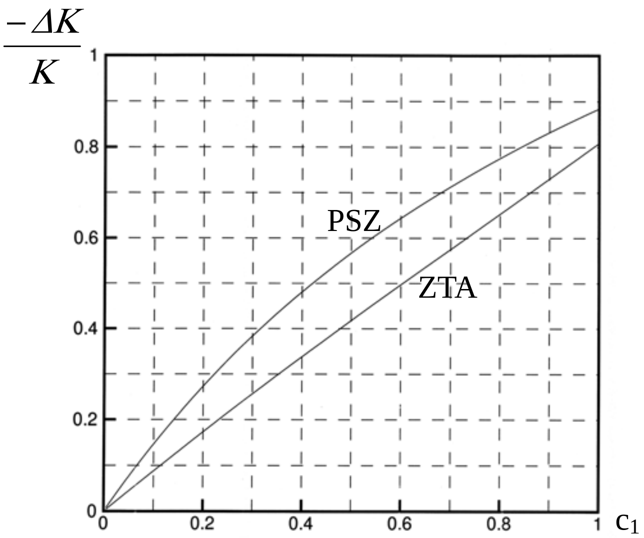

FIG.7 The normalized toughness increment with volume concentration in PSZ and ZTA materials, where K=c_0K_0+c_1K_1.

球形介質相變引起殘留應力之韌性強化

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關鍵詞：殘留應力、破壞韌性、非均質

摘要

提出一種根據均值法之微觀力學理論來推導相變所產生之殘留應力，並求出相變區之材料母體的應力。當介質材料體積比超過 30%時，和連體理論所得之殘留應力比較，本文所推導的殘留應力更適合用在非均質固體材料上。複合材料之母體與介質有不同彈性模數時，由連體理論計算之殘留應力必須作修正，而該修正係數也已經計算求得。經由殘留應力引起強化韌性之結果顯示，體積膨脹之相變和比母體彈性模數弱之介質材料能夠更有效強化材料韌性。具有球形介質相變之韌性變化公式也已用明確式表示。

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