剛性顆粒複合材料之韌性及彈性模數
The Toughness and Elastic Moduli of Rigid-Reinforced Composites

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摘　要

由於彈性模數的變化，使得含剛性顆粒之複合材料的裂縫尖端韌性能夠用理論解求出。當剛性球形介在物加入具等向性性質之母材時，在複合材料已存在缺陷或裂縫附近的破壞韌性將會降低，使得材料更具脆性。結果顯示，加入剛性物質之複合材料的彈性模數增加率會比其韌性之減少率還要大。若介在物含量爲30%，則母材之柏松比約在 0.34 時，所獲得的韌性減少率爲最少。有關含剛性介在物之複合材料的韌性脆化及彈性模數之明確公式，也已推導並且表示在本文裏。

關鍵詞：破壞韌性，剛性介在物，彈性模數。

Abstract

The crack-tip toughness of composite materials with rigid particles is evaluated theoretically because of the change of the effective elastic moduli. The fracture toughness in the vicinity of the main crack reduces as rigid spherical inclusions are employed within the isotropic matrix. The composites become more brittle. The results show that the ratio of the elastic moduli increment is greater than that of the toughness reduction. The most effective toughness enhancement in the rigid-reinforced composites is found if the Poisson ratio of the matrix approximately reaches 0.34 at c_1=0.3. The explicit forms of the toughness change and the effective elastic moduli of the rigid-reinforced composites are also derived and shown here.

Keywords: fracture toughness, rigid inclusion, elastic moduli

1. Introduction

It is very important to strengthen the materials in engineering. The elastic moduli of the composite will increase when the material composes of the harder inclusions and the softer matrix. However, this leads to the reduction of the material toughness. To find the relation between the strength and the toughness of the composites is interested to the material designers. As one knows, the effective elastic moduli of the composite depend on the material properties of the constituents, the shapes and the volume fraction of the inclusion [1-2], and the material toughness, too [3]. Here, one mainly concentrates on the toughness change due to the moduli enhancement of a rigid-reinforced composite. The rigid-reinforced composite consists of the isotropic matrix and the rigid inclusions, where the inclusions are uniformly dispersed within the matrix. The bulk and shear moduli of rigid inclusions go to infinity as compared with those of the matrix in the text. Throughout this work the matrix will be referred to as phase 0, and the inclusion as phase 1, with the volume fractions c_0 and c_1, respectively. For simplification, only the spherical inclusions are taken into account.

2. Effective Elastic Moduli of the Rigid-Reinforced Composite

The rigid-reinforced composite is isotropic when the inclusions are of 3-dimentional random orientations. Only two independent elastic constants, for instance, or the bulk modulus κ and the shear modulus μ exist in this composite. There are many theoretical approaches [4-6] to determine the effective moduli of the rigid-reinforced composite as $\kappa_1 \to \infty$ and $\mu_1 \to \infty$. One uses the Eshelby-Mori-Tanaka method [7-8] to evaluate those effective elastic moduli. Detailed exposition of this method, for example, can be found in Weng [9]. The explicit forms of effective moduli of the rigid-reinforced composite, then, can be determined from the results in Pan and Weng [10]. After some simple but lengthy calculations, the results containing rigid spherical inclusions are shown as follows.

1. Effective Bulk Modulus: κ

$$\frac{\kappa}{\kappa_0} = \frac{3\kappa_0 + 4\mu_0 c_1}{3\kappa_0(1-c_1)} \tag{1}$$

where the bulk and shear modulus are denoted as κ_0 and μ_0 respectively.

2. Effective Shear Modulus: μ

$$\frac{\mu}{\mu_0} = \frac{6(\kappa_0 + 2\mu_0\} + (9\kappa_0 + 8\mu_0)c_1}{6(\kappa_0 + 2\mu_0)(1-c_1)} \tag{2}$$

3. Effective Poisson's Ratio: ν

$$\nu = \frac{3(\kappa_0 + 2\mu_0)(3\kappa_0 - 2\mu_0) + \mu_0(3\kappa_0 + 16\mu_0)c_1}{6(\kappa_0 + 2\mu_0)(3\kappa_0 + \mu_0) + \mu_0(33\kappa_0 + 56\mu_0)c_1} \tag{3}$$

Obviously, the above results own the relations of $\kappa = \kappa_0$, $\mu = \mu_0$ and $\nu = \nu_0 = (3\kappa_0 - 2\mu_0)/(6\kappa_0 + 2\mu_0)$ when no inclusions happen within the matrix or $c_1 \to 0$, where ν_0 is the Poisson ratio of the matrix. This coincides with the real material system. Meanwhile, the effective elastic moduli of the rigid-reinforced composite depend on the material properties of the matrix and the volume concentration of the inclusions.

3. Toughness Change

From Eqs.(1) and (2), one finds that the rigid-reinforced composite always reveal the behavior of $\kappa \geq \kappa_0$ and $\mu \geq \mu_0$. Then, the increase of effective elastic moduli results in the toughness loss of the composite by instinct. The material toughness can be measured by the stress intensity factor. Let K_0 represent the stress intensity factor of the intact matrix, and K_{tip} do the stress intensity factor near the main crack-tip inside the composite. For a plane-strain condition and under the Mode I loading, the overall inhomogeneity toughness change of the isotropic medium is of the form [3].

$$\frac{K_{tip}}{K_0} = f\sqrt{g} \tag{4}$$

where

$$f = \frac{24pk_1(1-2\nu_0)(1+\nu_0)+12(1-\nu_0)}{p(1-2\nu_0)(1+\nu_0)+12(1-\nu_0)} \tag{5}$$

$$g = \frac{(1-\nu_0)[2(p-q)(1+\nu_0)-3(1+p)]}{(1+q)[(q-4p-3)(1-\nu_0)+2(p-q)]} \tag{6}$$

and the contour factor k_1 is 1/24 and 0.0072 under the stationary crack and the steadily growing crack condition, respectively. The parameter f is equal to one while the main crack is stationary. The material constant p and q have been found in the general case and shown in [10]. By substituting the Eshelby S-tensor [11] with the spherical shape of the inclusions into p and q, the final results are in the following.

$$p = -\frac{3c_1(1-\nu_0)}{1+\nu_0+2c_1(1-2\nu_0)} \tag{7}$$

$$q = -\frac{15c_1(1-\nu_0)}{8-10\nu_0+c_1(7-5\nu_0)} \tag{8}$$

Since the material constant p and q are found, the closed solution of the toughness change of the rigid-reinforced composite, by substituting Eq. (7) and Eq. (8) into Eq. (5) and Eq. (6), is determined with

$$f = \frac{4\{1+\nu_0+2c_1(1-2\nu_0)[1-3k_1(1+\nu_0)]\}}{4(1+\nu_0)+c_1(1-2\nu_0)(7-\nu_0)} \tag{9}$$

$$g=\frac{(8-10\nu_0)[-8+10\nu_0-2c_1(10-23\nu_0+15\nu_0^2)]-c_1^2(1-2\nu_0)(91-170\nu_0+75\nu_0^2)}{4(4-5\nu_0)[-4+5\nu_0-5c_1(1-2\nu_0)](1-c_1)} \tag{10}$$

The explicit forms in Eqs. (4), (9) and (10) show that the toughness change of the composite containing rigid spherical inclusions only depends on the Poisson ratio of the matrix and the volume fraction of the inclusions. Noted that the rigid-reinforced composite always have the behavior of $K_{tip}/K_0 \geq 1$. This leads to the composite more brittle.

4. Numerical Results

The effective toughness (measured toughness), $K_m = K_0 - \Delta K$, of the composite is less than K_0 because of the existence of the rigid inclusions with the toughness change ΔK. In other words, the crack-tip toughness $K_{tip} = K_0 + \Delta K$. The composite material become more brittle if $K_{tip} > K_0$ [12].

The propagation condition of a main crack will affect the material toughness depending on k_1 shown in Eq, (9). The normalized toughness vs the volume fraction c_1 is plotted in Fig. (1). It seems that the rigid-reinforced composite under the steadily growing crack condition is more toughened than under the stationary crack one. This behavior actually has been observed in many real material systems [13].

Due to the fact that the toughness behavior of the stationary crack and the steady-state propagating crack is similar to each other, one may only discuss the material properties for a steadily growing crack in the following. The relations between the toughness and the effective moduli with $\nu_0 = 0.1$, 0.3 and 0.5 are depicted in Fig. (2)-(4). All conclude the result that the ratio of the normalized moduli is greater than that of the normalized toughness. For instance in Fig. (2), $\kappa/\kappa_0 = 2.2$ and $\mu/\mu_0 = 1.7$ at c_1=0.3, but K_{tip}/K_0=1.4 only. In the other hand, the loss ratio of the toughness is less than the strengthening ratio in the rigid-reinforced composite. The effective bulk modulus decreases as ν_0 increases from 0 to 0.5, and this is totally reversed for the shear modulus shown in Fig. (2)-(4).

To the end, one examines the toughness change due to the effect of the matrix's Poisson ratio. The normalized toughness vs the Poisson ratio of the matrix at c_1=0.3 is depicted in Fig. (5). The rigid-reinforced composite is more brittle if the Poisson ratio of the matrix approaches to 0 or 0.5. The minimum toughness reduction of the composite occurs at $\nu_0 = 0.34$. This result implies that we can obtain the optimum material properties by choosing the appropriate Poisson ratio of the matrix.

5. Conclusions

The overall quantity of the material properties in the rigid-reinforced composite is investigated. The toughness and the elastic moduli of the composite are all shown in the explicit forms. The toughness of the composite is more than a half of that of the matrix if the volume fraction of the inclusions $c_1 = 0.5$ shown in Fig. (2)-(4). However, the strengthening of the effective elastic moduli is more than twice of the elastic modulu of the matrix. As the volume fraction of the rigid inclusions increases after c_1=0.4, the influence of the toughness and the effective moduli of the composite become large. The Poisson ratio of the matrix dominates the toughness change of the reinforced composite if the volume concentration of the inclusions is constant.

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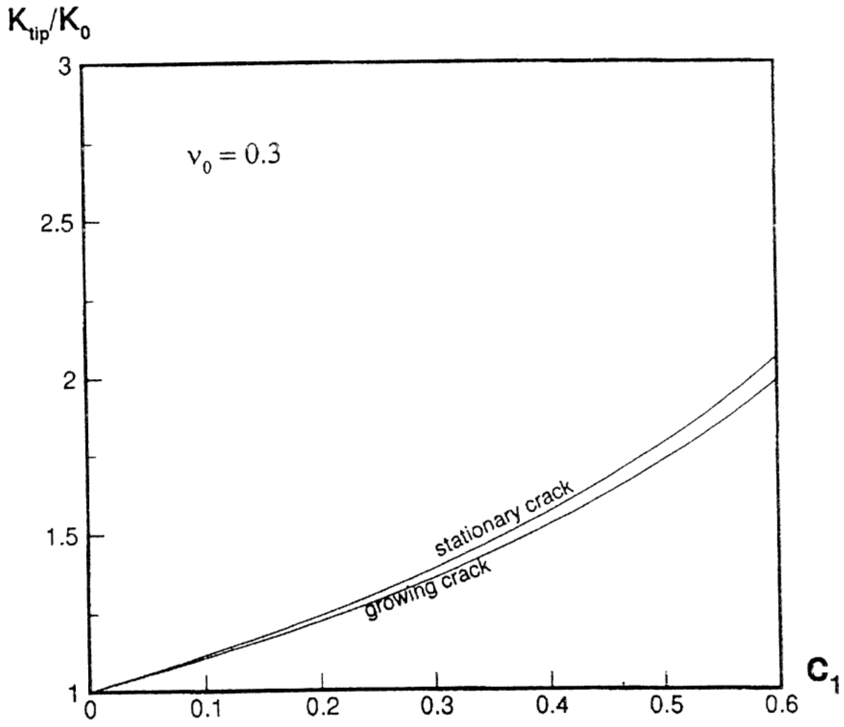

Fig. 1 The normalized toughness vs the volume fraction of the inclusions at $\nu_0 = 0.3$.

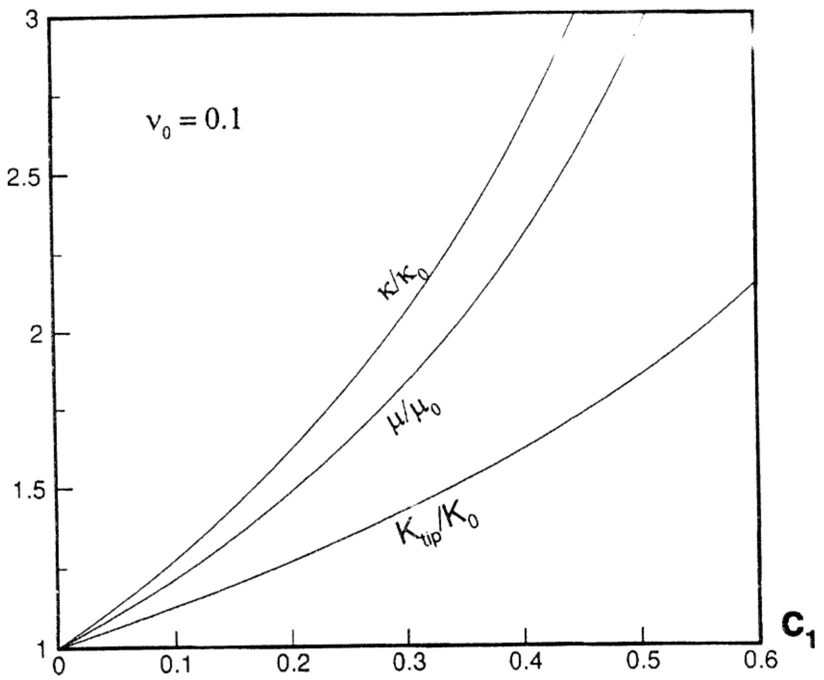

Fig. 2 The relation of the elastic moduli and the toughness at $\nu_0 = 0.1$.

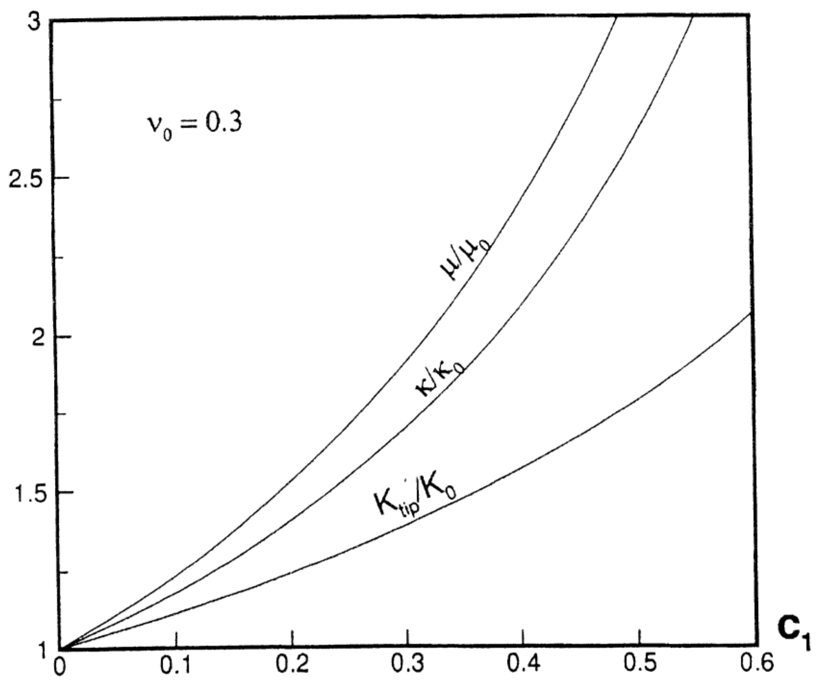

Fig. 3 The relation of the elastic moduli and the toughness at $\nu_0 = 0.3$.

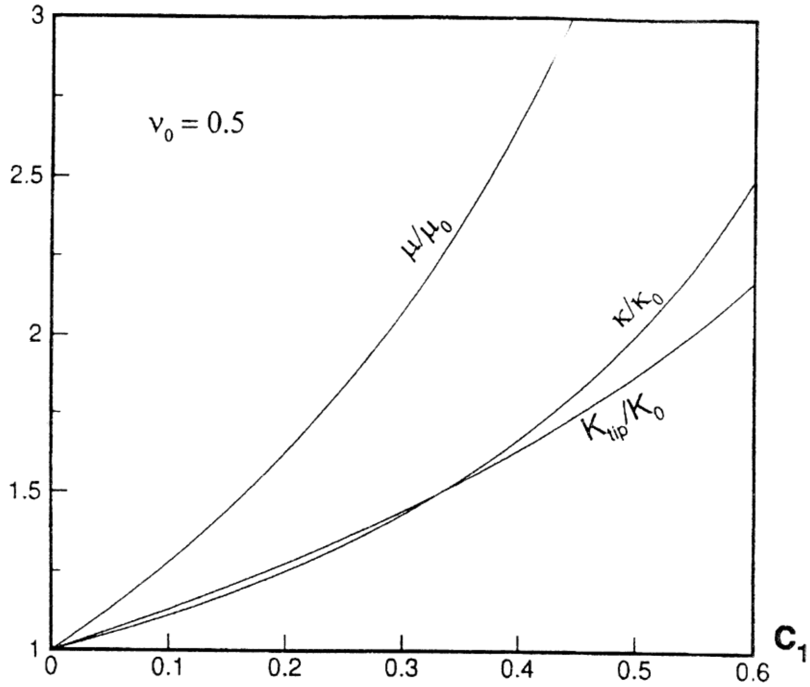

Fig. 4 The relation of the elastic moduli and the toughness at $\nu_0 = 0.5$.

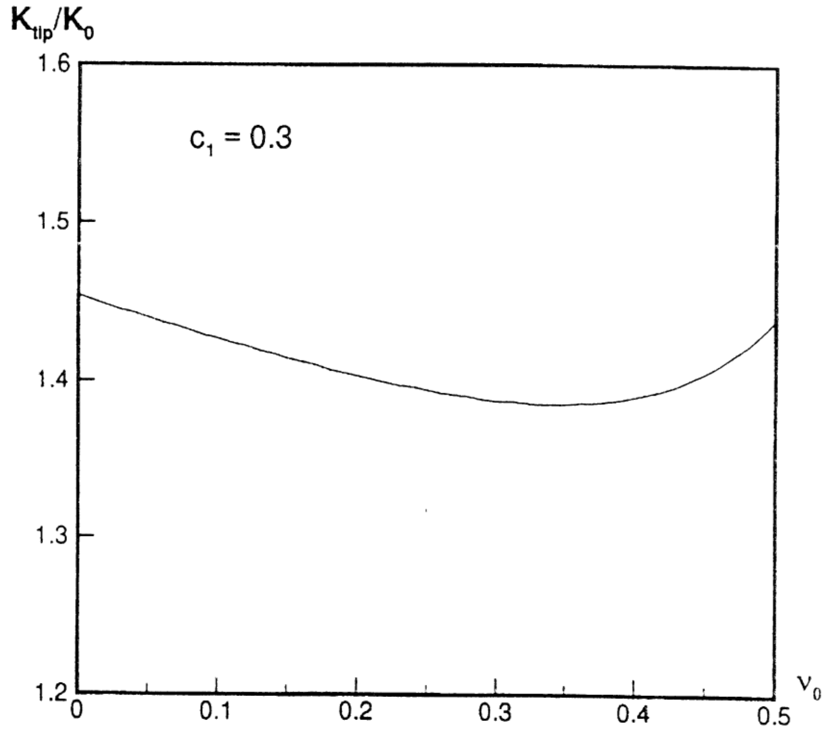

Fig. 5 The normalized toughness vs the Poisson ratio of the matrix at $c_1 = 0.3$.

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