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A Micromechanical Theory for the Determination of the stress-strain Relation of Cement-Matrix Composites

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A Micromechanical Theory for the Determination of the Stress-Strain Relation of Cement-Matrix Composites

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Abstract

A micromechanical theory is presented to simulate the stress-strain relation of cement-matrix composites. Based on the secant modulus of the matrix, the moduli of the composite material are found. The stress-strain relation of the composites depends on the material properties of the constituents, the volume fraction and the shape of the inclusions. As the stress-strain behavior of the matrix is known, the theoretical stress-strain curve of the composite will be determined. The elastic moduli of the concrete with the disk-like aggregates are stiffer than those with the spherical one if the aggregate is harder. When the volume concentration of the aggregates increases, the strain of concrete reduces because of the harder aggregates. The results also show the explicit form of the secant moduli of concrete. The derived formula is suitable for concrete with the three-dimensional randomly oriented aggregates.

1. Introduction

For the full stress-strain curve of concrete between the origin and at failure, the behavior actually is non-linear, or the so-called elastic-plastic. From the ACI Code, the elastic moduli of concrete rely on its unit weight and the fracture strength f_c. Meanwhile, the stress-strain curve of concrete can find the elastic moduli of the materials. The elastic Young modulus of concrete is the slope between the stress at the strain 0.00005 and the stress equaling to $0.4f_c$.

There are many models to predict the stress-strain relationship of concrete [1-7]. Most of them are all in terms of the fracture strength of the concrete. However, the stress-strain curves may vary at the same concrete strength. It seems that the strength f_c of concrete is not a prime factor to control the moduli. To neglect the strength factor different from previous models, the effective elastic moduli of the composites are found in many literatures [8-11] with elastic moduli of the matrix and the inclusion. The concept of elastic constraint was also present to estimate the overall elastoplastic stress-strain relations of dual-phase metals [12]. An energy approach with the secant moduli method, a more accurate method, was also proposal to determinate the plastic and viscoplastic behavior of the composite materials [13-14]. A micromechanics concept with the secant moduli, now, is chosen to calculate the stress-strain of concrete for the first time.

The aim of this project is to determine the stress-strain relation of the concrete theoretically. Concrete as a two-phase composite consists of the cement matrix and the aggregate (inclusion). The moduli of the aggregate are assumed to be constant and those of the cement are elastic-plastic under the loadings. Because of the elastic-plastic behavior of the cement, the stress-strain curve of concrete becomes non-linear depending on the volume fraction of the aggregate.

2. Micromechanics Theory

We shall use the Mori-Tanaka [15] method to determine the effective moduli of the two-phase composites. Detailed exposition of this method based on Eshelby's [16] equivalent transformation strain (or eigenstrain) ε^* can be found in [17]. In the two-phase system the inclusions will be referred to as phase 1, and the matrix as phase 0. The volume fraction of the r-th phase will be denoted by c_r. Then the elastic moduli of the r-th phase will be written as L_r, with the bulk and shear components $L_r = (3\kappa_r, 2\mu_r)$.

With the three-dimensional randomly oriented ellipsoidal inclusions the composite as a whole is isotropic, characterized by its effective bulk and shear moduli, or κ and μ respectively. Following the eigenstrain approach, one obtains the effective bulk and shear moduli are found to be

$$\kappa = \kappa_0 /[1 + c_1(p_2/p_1)], \quad (1)$$

$$\mu = \mu_0 /[1 + c_1(q_2/q_1)], \quad (2)$$

where the material constants p_1, p_2, q_1 and q_2 are

$$p_1 = 1 + c_1[b_1 + 2(b_2 + b_3 + b_4 + b_5)]/3,$$

$$p_2 = (a_{11} + a_{12} + a_{13} + a_{21} + a_{22} + a_{23} + a_{31} + a_{32} + a_{33})/3,$$

$$q_1 = 1 + c_1[2(b_1 - b_2 - b_3) + 7b_4 - 5b_5 + 6b_6]/15,$$

$$q_2 = [3(b_{12} + b_{13} + b_{23}) + 2(a_{11} + a_{22} + a_{33}) - (a_{12} + a_{13} + a_{21} + a_{23} + a_{31} + a_{32})]/15.$$

The constants a_{ij}, b_{ij} and b_i depend on the moduli of the constituents and Eshelby's S tensor for an ellipsoidal inclusion; their values are listed in Appendix.

The explicit form, for example, of the effective bulk and shear moduli with the spherical inclusions can be found from Eq. (1) and Eq. (2) to be

$$\kappa = [\kappa_0(3\kappa_1 + 4\mu_0) - 4c_1\mu_0(\kappa_0 - \kappa_1)] / [3\kappa_1 + 4\mu_0 + 3c_1(\kappa_0 - \kappa_1)] \quad (3)$$

$$\mu = \mu_0[5\mu_1(3\kappa_0 + 4\mu_0) + c_0(\mu_0 - \mu_1)\cdot (9\kappa_0 + 8\mu_0)]/[5\mu_0(3\kappa_0 + 4\mu_0) - 6c_0(\mu_0 - \mu_1)(\kappa_0 + 2\mu_0)] \quad (4)$$

where $c_0 = 1 - c_1$ is the volume fraction of the matrix.

Now, the secant Young modulus, referred as $E_0(\varepsilon)$, of the matrix shown in Fig.1 is non-linear, and it relies on the applied stress or applied strain, but the Poisson ratio of the matrix remains constant. Fig. 2 is the modulus of the inclusion E_1 and its value is constant. The relations of the material constants are

$$E = 9\kappa\mu/(3\kappa + \mu) \quad (5)$$

$$\nu = (3\kappa - 2\mu)/(6\kappa + 2\mu) \quad (6)$$

where ν is denoted as the Poisson ratio of the material.

To estimate the effective Young modulus $E(\varepsilon)$ of the composite, we replace $E_0(\varepsilon)$ instead of E_0 in Eq. (1) and Eq. (2), where E_0 is the elastic Young modulus of the matrix.

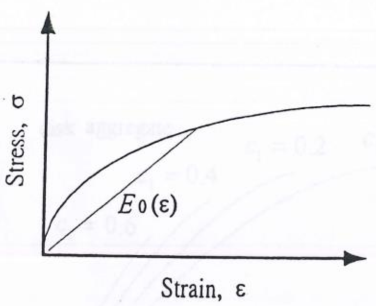

Fig. 1 Schematic representation of the secant Young modulus of the matrix.

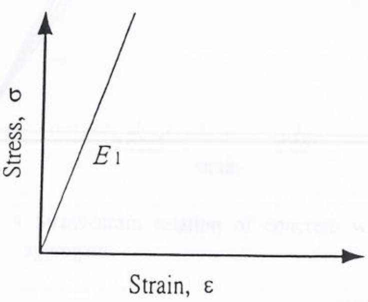

Fig. 2 Schematic representation of the elastic Young modulus of the inclusions.

3. Numerical Results

To simulate the stress-strain curve of concrete, we choose the cement to be the matrix, and its secant Young modulus is assumed to be [18]

$$E_0(\varepsilon) = (2 - e^{a\varepsilon/\varepsilon_u})E_0 . \quad (7)$$

The strain ε in Eq. (7) is the strain at any stress state before failure and ε_u the strain at fracture f_c, respectively. The parameter a is chosen as

$$a = -0.00135\, E_0^2 + 0.08652\, E_0 - 0.23359$$

This parameter can be found if the initial elastic Young modulus of the cement E_0 is known.

The material properties of the constituents before loading are [18-19]

aggregate: $E_1 = 40GPa$ and $\nu_1 = 0.1$

cement: $E_0 = 27.26GPa$ and $\nu_0 = 0.22$

Assuming that the Poisson ratio of the cement and aggregate remains constant during the loading. When a uniaxial compressive load applies to concrete, the strain ε_0 exists. The effective strain of concrete $\bar{\varepsilon}$ calculated from the mean-field approach is [17]

$$\bar{\varepsilon} = \varepsilon_0 + c_1\varepsilon^* . \quad (8)$$

The eigenstrain ε^* is in terms of the applied strain ε_0 and the material properties we neglect its results here.

The procedures to simulate the stress-strain curve of concrete begin at infinitesimal ε_0. This strain allows us to calculate the secant modulus in Eq. (7), so as to find the effective secant modulus of concrete in Eq. (1) and Eq. (2). Meanwhile the effective strain of concrete is determined from Eq. (8). Then the stress-strain curve of concrete is found by

$$\sigma(\varepsilon) = E(\varepsilon)\cdot\bar{\varepsilon} . \quad (9)$$

The stress-strain relations of concrete with the spherical aggregates and with the disk-like ones are depicted in Fig. 3 and Fig. 4, where the volume fraction of aggregates c_1 is 0.0, 0.2, 0.4 and 0.6, respectively. In Fig. 3, the effective secant Young modulus $E_0(\varepsilon)$ is stiffer than that of the cement because the harder aggregate used here. When the volume concentration of aggregates increases, the effective strain of concrete reduces but the moduli become stronger. Concrete with the disk aggregates has the same results in Fig. 4. By comparing Fig.3 and Fig. 4, we find that the moduli of concrete with the disk aggregate are stiffer than those with the spherical ones. Therefore, the shape and the volume fraction of the aggregate affect the stress-strain relation of the concrete.

4. Conclusions

According to the micromechanics approach, the stress-strain relationship of concrete is determined. The secant moduli of concrete depend on the shape of the aggregate, the material properties and the volume fraction of the constituents, not the fracture strength f_c and the unit weight of concrete. Once the stress-strain curve of the cement knows, the stress-strain relation of concrete will determine. This method is simple enough to engineers. However, we should note that this approach does not use to estimate the fracture strength f_c of concrete.

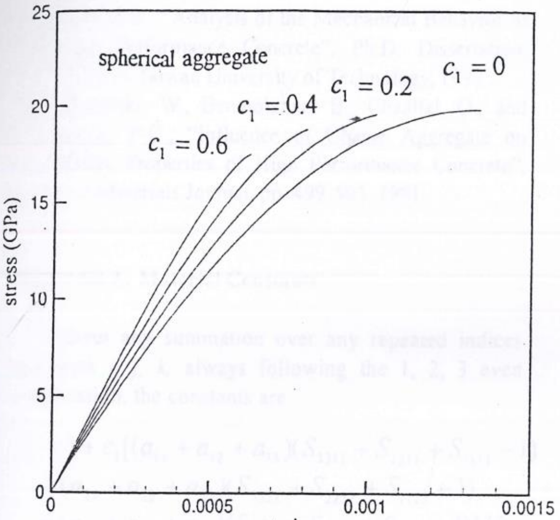

Fig. 3 Stress-strain relation of concrete with the spherical aggregate.

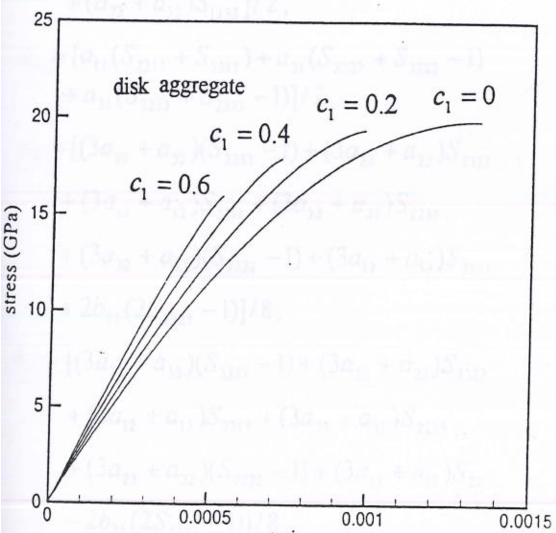

Fig. 4 Stress-strain relation of concrete with the disk aggregate.

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Appendix A: Material Constants

Without any summation over any repeated indices and with i, j, k, always following the 1, 2, 3 even permutation, the constants are

$$p_1 = 1 + c_1[(a_{11} + a_{12} + a_{13})(S_{3311} + S_{2211} + S_{1111} - 1) + (a_{21} + a_{22} + a_{23})(S_{3322} + S_{2222} + S_{1122} - 1) + (a_{31} + a_{32} + a_{33})(S_{3333} + S_{2233} + S_{1133} - 1)]/3,$$

$$p_2 = (a_{11} + a_{12} + a_{13} + a_{21} + a_{22} + a_{23} + a_{31} + a_{32} + a_{33})/3,$$

$$b_1 = a_{11}(S_{1111} - 1) + a_{21}S_{1122} + a_{31}S_{1133},$$

$$b_2 = [(a_{12} + a_{13})(S_{1111} - 1) + (a_{22} + a_{23})S_{1122} + (a_{32} + a_{33})S_{1133}]/2,$$

$$b_3 = [a_{11}(S_{2211} + S_{3311}) + a_{21}(S_{2222} + S_{3322} - 1) + a_{31}(S_{3333} + S_{2233} - 1)]/2,$$

$$b_4 = [(3a_{33} + a_{32})(S_{3333} - 1) + (3a_{23} + a_{22})S_{3322} + (3a_{13} + a_{12})S_{3311} + (3a_{32} + a_{33})S_{2233} + (3a_{22} + a_{23})(S_{2222} - 1) + (3a_{12} + a_{13})S_{2211} + 2b_{23}(2S_{2323} - 1)]/8,$$

$$b_5 = [(3a_{32} + a_{33})(S_{3333} - 1) + (3a_{22} + a_{23})S_{3322} + (3a_{12} + a_{13})S_{3311} + (3a_{33} + a_{32})S_{2233} + (3a_{23} + a_{22})(S_{2222} - 1) + (3a_{13} + a_{12})S_{2211} - 2b_{23}(2S_{2323} - 1)]/8,$$

$$b_6 = [b_{12}(2S_{1212} - 1) + b_{13}(2S_{1313} - 1)]/2,$$

$$b_{12} = (1 - \mu_1/\mu_0)/[1 + 2S_{1212}(\mu_1/\mu_0 - 1)],$$

$$b_{13} = (1 - \mu_1/\mu_0)/[1 + 2S_{1313}(\mu_1/\mu_0 - 1)],$$

$$b_{23} = (1 - \mu_1/\mu_0)/[1 + 2S_{2323}(\mu_1/\mu_0 - 1)],$$

$$a_{ii} = [3(\kappa_1 - \kappa_0)(\mu_1 - \mu_0)^2(S_{jjjj}S_{kkkk} - S_{jjkk}S_{kkjj}) - (\mu_1 - \mu_0)(\kappa_1\mu_0 - \kappa_0\mu_1)(S_{jjjj} + S_{kkkk} - S_{jjkk} - S_{kkjj}) + 3\mu_0(\kappa_1 - \kappa_0)(\mu_1 - \mu_0)(S_{jjjj} + S_{kkkk}) + 3\kappa_0\mu_0(\mu_1 - \mu_0) + \mu_0(\kappa_1\mu_0 - \kappa_0\mu_1)]/A,$$

$$a_{ij} = [3(\kappa_1 - \kappa_0)(\mu_1 - \mu_0)^2(S_{iikk}S_{kkjj} - S_{iijj}S_{kkkk}) - (\mu_1 - \mu_0)(\kappa_1\mu_0 - \kappa_0\mu_1)(S_{iikk} + S_{kkjj} - S_{iijj} - S_{kkkk}) - 3\mu_0(\kappa_1 - \kappa_0)(\mu_1 - \mu_0)S_{iijj} + \mu_0(\kappa_1\mu_0 - \kappa_0\mu_1)]/A,$$

$$A = (\mu_1 - \mu_0)(\kappa_1\mu_0 - \kappa_0\mu_1)[S_{3333}(S_{1111} + S_{2222} - S_{1122} - S_{2211}) + S_{3322}(S_{1133} + S_{2211} - S_{1111} - S_{2233}) + S_{3311}(S_{1122} + S_{2233} - S_{1133} - S_{2222}) + S_{2211}(S_{1133} - S_{1122}) + S_{2222}(S_{1111} - S_{1133}) + S_{2233}(S_{1122} - S_{1111})] + 3(\kappa_1 - \kappa_0) \cdot (\mu_1 - \mu_0)^2[S_{3333}(S_{1122}S_{2211} - S_{1111}S_{2222}) + S_{3322}(S_{1111}S_{2233} - S_{1133}S_{2211}) + S_{3311} \cdot (S_{1133}S_{2222} - S_{1122}S_{2233})] + 3\mu_0(\kappa_1 - \kappa_0) \cdot (\mu_1 - \mu_0)(S_{1122}S_{2211} + S_{1133}S_{3311} + S_{2233}S_{3322} - S_{1111}S_{2222} - S_{2222}S_{3333} - S_{3333}S_{1111}) - \mu_0(\kappa_1\mu_0 - \kappa_0\mu_1)(S_{1111} + S_{1122} + S_{1133} + S_{2211} + S_{2222} + S_{2233} + S_{3311} + S_{3322} + S_{3333}) - 3\kappa_0\mu_0 \cdot (\mu_1 - \mu_0)(S_{1111} + S_{2222} + S_{3333} - 1) - 3\kappa_0\mu_0\mu_1,$$

Appendix B: Eshelby's S_{ijkl} Tensor

Eshelby's tensor is in terms of the Poisson ratio of the matrix ν_0 and the shape of the inclusion. It has a property of $S_{ijkl} = S_{jikl} = S_{ijlk}$. The three radii of an ellipsoid are a_1, a_2 and a_3, where $a_1 > a_2 > a_3$. The properties of an ellipsoid within the matrix can be described by twelve independent components of the Eshelby tensor, which are

$$S_{1111} = [3(a_1 I_{11}) + (1 - 2\nu_0)I_1]/[8\pi(1 - \nu_0)]$$

$$S_{1122} = [a_2^2 I_{12} - (1 - 2\nu_0)I_1]/[8\pi(1 - \nu_0)]$$

$$S_{1133} = [a_3^2 I_{13} - (1 - 2\nu_0)I_1]/[8\pi(1 - \nu_0)]$$

$$S_{1212} = [(a_1^2 + a_2^2)I_{12} + (1 - 2\nu_0)(I_1 + I_2)]/[16\pi(1 - \nu_0)]$$

and all other non-zero components are obtained by the cyclic permutation of (1, 2, 3). The S_{ijkl} tensor which can not be calculated from above are zero. The I_i and I_{ij} components are

$$I_1 = (4\pi a_1 a_2 a_3)[F(\theta, k) - E(\theta, k)] / [(a_1^2 - a_2^2)(a_1^2 - a_3^2)^{1/2},$$

$$I_3 = (4\pi a_1 a_2 a_3)[a_2(a_1^2 - a_3^2)^{1/2}/(a_1 a_3) - E(\theta,k)]$$
$$/[(a_2^2 - a_3^2)(a_1^2 - a_3^2)^{1/2},$$

$$I_2 = 4\pi - I_1 - I_3 \ ,$$

$$I_{11} = [4\pi a_2^2(a_1^2 - a_3^2) + a_1^2(a_2^2 + 2a_3^2 - 3a_1^2)I_1$$
$$+ a_1^2(a_3^2 - a_2^2)I_3]/[3a_1^2(a_1^2 - a_2^2)(a_3^2 - a_1^2)],$$

$$I_{22} = 4\pi\{[a_1^2 a_3^2 - a_2^2(2a_1^2 + 2a_3^2 - 3a_2^2)]$$
$$+ a_2^2(a_1^2 + 2a_3^2 - 3a_2^2)I_1 + a_2^2(a_3^2 + 2a_1^2 - 3a_2^2)I_3\}$$
$$/[3a_2^2(a_1^2 - a_2^2)(a_3^2 - a_2^2)],$$

$$I_{33} = [4\pi a_2^2(a_3^2 - a_1^2) + a_3^2(a_1^2 - a_2^2)I_1$$
$$+ a_3^2(a_2^2 + 2a_1^2 - 3a_3^2)I_3]/[3a_3^2(a_1^2 - a_3^2)(a_3^2 - a_2^2)]$$

$$I_{12} = (4\pi - 2I_1 - I_3)/(a_1^2 - a_2^2),$$

$$I_{13} = (I_3 - I_1)/(a_1^2 - a_3^2),$$

$$I_{23} = (I_1 + 2I_3 - 4\pi)/(a_2^2 - a_3^2)$$

and the elliptic integrals of the first and the second kind are

$$F(\theta,k) = \int_0^\theta [1/(1 - k^2\sin^2 w)^{1/2}]\cdot dw,$$

$$E(\theta,k) = \int_0^\theta (1 - k^2\sin^2 w)^{1/2}\cdot dw$$

where

$$\theta = \sin^{-1}(1 - a_3^2/a_1^2)^{1/2},$$
$$k = [(a_1^2 - a_2^2)/(a_1^2 - a_3^2)]^{1/2}.$$