Influence of the Constituents on the Elastic Moduli and the Strength of Concrete

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Abstract

This research is to examine the elastic Young modulus of concrete predicted in ACI Code. By measuring each compositions of concrete, the experimental material properties of concrete at the age of 14, 28 and 56 days are found. The results show that the theoretical M-T moduli are close to the experimental results, but not those predicted from ACI Code. The moduli of concrete calculated from ACI Code and compared to the experimental data are only good for the aggregates at least reaching 70% volume fraction of fine and coarse aggregates. The elastic modulus of concrete estimated from the fracture stress is not a proper way, and will overestimate when the volume concentration of aggregates is low. The proposed theory seemly is suitable for evaluating the elastic loduli of concrete.

1. Introduction

Concrete indeed is a composite material consisting of cement paste (or binders), fine and coarse aggregates. Many factors such as the mix proportion and the material properties of the constituents dominate the strength and the elastic moduli of concrete. The upper bound and lower bound of the effective elastic moduli can be found, depending on the volume fraction and the elastic moduli of the compositions, under a prescribed displacement and prescribed traction respectively [1]. This means that the material properties of the compositions theoretically control the elastic moduli of the composite. Meanwhile, many theoretical models and experimental results [2-7] also show the elastic moduli of concrete rely on the material properties of cement paste and the aggregates. Nevertheless, the estimation of elastic Young modulus in concrete commonly used is according to ACI Code as

$$E_c = 4730\sqrt{f_c^{'}} \quad (\text{N/mm}^2) \qquad (1)$$

where E_c=elastic Young modulus of concrete and $f_c^{'}$= fracture stress at the 28th day. It seems that ACI Code mainly considers the fracture stress as a dominated factor to calculate the elastic Young modulus of concrete. Obviously, these are different views between in practices and in engineering uses

The aim of this paper is to analyze the elastic moduli of concrete obtained from the theoretical models, the experiments and ACI Code, and try to find which one is reliably predicted method. The presented theory here is based on the mean-field approach [8-10], and takes concrete as a two-phase composite with the mortar (matrix) and the coarse aggregate (inclusion). The coarse aggregate is assumed to be perfectly bonded in the mortar, and has the same shape but with different sizes. To cover all possible shapes of the aggregate from fiber, sphere and disc, the inclusion shape is taken as an ellipsoid. In the two-phase system the inclusions (coarse aggregates) will be referred to as phase 1, and the matrix (mortar) as phase 0. The volume fraction of the *r*-th phase will be denoted by c_r. Then the elastic moduli of the *r*-th phase will be written as L_r, with the bulk and shear components $L_r = (3\kappa_r, 2\mu_r)$.

2. Experiments

We used Type I Portland cement, Ottawa standard sand satisfying ASTM C778 and the coarse aggregate from Lau-Lung River near Kaohsiung and Pingtung area to make concrete. The material properties of sand and coarse aggregate are shown in Table 1.

Table 1. Physical properties of fine and coarse aggregates.

Materials	Fine aggregate	Coarse aggregate
Specific gravity	2.65	2.61
Water contents (%)	0.11	0.90
SSD (%)	0.24	1.12
Particle size (mm)	0.3	4.75~9.5

In order to simplify the material parameters, the matrix consists of cement paste with w/c=0.44 (water/cement) and sands with 20% fixed volume fraction of concrete. Concrete was produced by changing the volume fraction of the inclusion (coarse aggregates) with c_1 =0, 0.1, 0.3 and 0.5, where c_1 = volume concentration of coarse aggregates. The mix design of concrete is also displayed in Table 2.

Table 2. Mix design of concrete (kg/m^3).

c_1	Water	Cement	Sand	Coarse aggregate
0	558	1269	635	0
0.1	488	1110	635	312
0.3	349	792	635	938
0.5	209	475	635	1560

The manufacture and the curing of concrete samples follow the procedures of CNS 1230 and CNS 1231 with specimen size 5 φ ×10cm and 10 φ ×20cm. The MTS machine was used to supply a compressive load during the testing. Before the test began, the samples had been preloaded 0.5kN~1.0kN to avoid the unnecessary electronic signals because of the sensitivity of the extensometer. We applied a uniaxial compressive stress to samples by following CNS 1232 with a constant strain 0.002mm/sec when the ages of concrete reach the 14^{th}, 28^{th} and 56^{th} day respectively, and tried to determine the fracture strength, the Poisson ratio ν and the elastic Young modulus E of concrete, where the elastic Young modulus calculated from ASTM C49. In the meantime, we took the cores of the coarse aggregate with 5 φ ×10cm specimen to find its elastic moduli.

After measuring the elastic Young modulus and the Poisson ratio of concrete, we use the relation of elastic moduli to calculate its bulk and shear modulus as follows.

$$\kappa = \frac{E}{3(1-2\nu)} \tag{2}$$

$$\mu = \frac{E}{2(1+\nu)} \tag{3}$$

where κ and μ are the bulk and the shear modulus of concrete individually. The experimental results of the bulk and shear moduli for mortar and the coarse aggregate are shown in Table 3.

Table 3. Elastic moduli of mortar and coarse aggregates.

The ages	κ_0 (*)	μ_0 (*)	κ_1 (*)	μ_1 (*)
14^{th} day	9.08	7.59	19.44	14.58
28^{th} day	10.29	8.37	19.44	14.58
56^{th} day	11.30	9.20	19.44	14.58

* the unit is GPa.

3. The M-T Moduli

Many predictions of the elastic moduli, based on the empirical formula and micromechanics models, have been proposed. In presence, we shall use the Mori-Tanaka [8] method to determine the effective moduli of the two-phase composites, or the so-called M-T moduli. Detailed exposition of this method based on Eshelby's [9] equivalent transformation strain (or eigenstrain) ε^* can be found in Weng [10].

With the three-dimensional randomly oriented ellipsoidal inclusions the composite as a whole is isotropic, characterized by its effective bulk and shear moduli, or κ and μ respectively. Following the eigenstrain approach [6], one obtains the explicit forms of the effective bulk and shear moduli to be

$$\kappa = \kappa_0 / [1 + c_1 (p_2 / p_1)], \tag{4}$$

$$\mu = \mu_0 / [1 + c_1 (q_2 / q_1)], \tag{5}$$

where the material constants p_1, p_2, q_1 and q_2 are

$$\begin{aligned} p_1 &= 1 + c_1[b_1 + 2(b_2 + b_3 + b_4 + b_5)]/3, \\ p_2 &= (a_{11} + a_{12} + a_{13} + a_{21} + a_{22} + a_{23} \\ &\quad + a_{31} + a_{32} + a_{33})/3, \\ q_1 &= 1 + c_1[2(b_1 - b_2 - b_3) + 7b_4 - 5b_5 + 6b_6]/15, \\ q_2 &= [3(b_{12} + b_{13} + b_{23}) + 2(a_{11} + a_{22} + a_{33}) \\ &\quad - (a_{12} + a_{13} + a_{21} + a_{23} + a_{31} + a_{32})]/15. \end{aligned} \tag{6}$$

The constants a_{ij}, b_{ij} and b_i depend on the moduli of the constituents and Eshelby's S tensor for an ellipsoidal inclusion; their values are listed in Appendices.

The explicit form of the effective bulk and shear moduli with the spherical inclusions, a special case, can be found from Eq. (4) and Eq. (5) to be

$$\kappa = [\kappa_0 (3\kappa_1 + 4\mu_0) - 4c_1\mu_0(\kappa_0 - \kappa_1)] / [3\kappa_1 + 4\mu_0 + 3c_1(\kappa_0 - \kappa_1)] \tag{7}$$

$$\begin{aligned} \mu = \mu_0[5\mu_1(3\kappa_0 + 4\mu_0) + c_0(\mu_0 - \mu_1)\cdot \\ (9\kappa_0 + 8\mu_0)]/[5\mu_0(3\kappa_0 + 4\mu_0) \\ - 6c_0(\mu_0 - \mu_1)(\kappa_0 + 2\mu_0)] \end{aligned} \tag{8}$$

where $c_0 = 1 - c_1$ is the volume fraction of the matrix. Eqs. (7)-(8) will be taken as the theoretical results of concrete.

4. Results and Discussion

The fracture stress of concrete with different volume concentration of coarse aggregate at aged 14, 28 and 56 days are shown in Fig.1, and the strength decreases as the coarse aggregate increases.

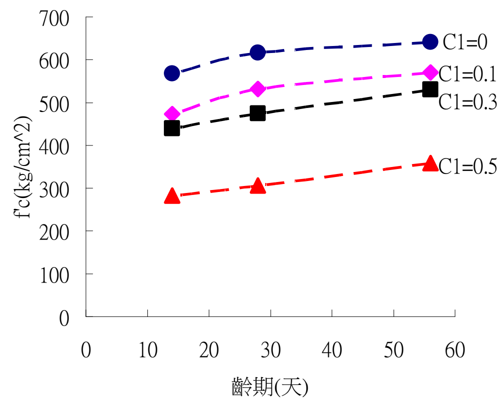

Fig. 1 Fracture stress of concrete.

The elastic Young moduli of concrete increase with increasing different volume fraction and the ages are depicted in Fig. 2, where the dotted lines are the experimental data and the solid lines are M-T moduli calculated from Eqs. (2)-(3) and Eqs. (7)-(8). In Fig.3, the increasing ratios of Young modulus E/E_0 gradually decrease as the ages are old, but gain linearly as the volume fraction of aggregates rises, where E_0=Young modulus of mortar.

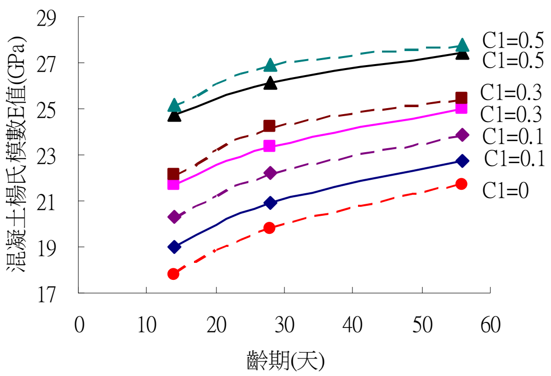

Fig. 2 Elastic Young modulus of concrete.

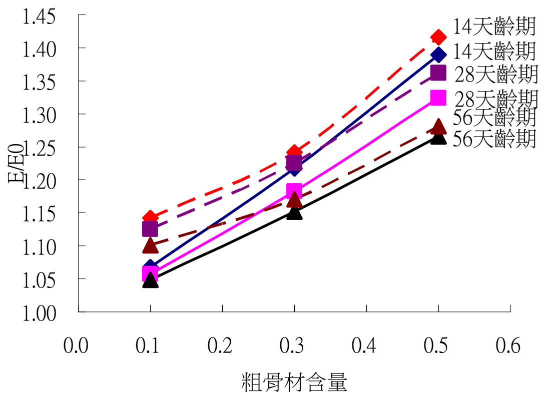

Fig. 3 Increasing ratio of Young modulus.

Fig. 4 is the comparisons between the experiments and the M-T moduli of the shear modulus of concrete. The variations between them become large when c_1 increases, especially at c_1 =50%. The ratios of the shear modulus also decrease when the ages are old, like Fig. 3, shown in Fig.5.

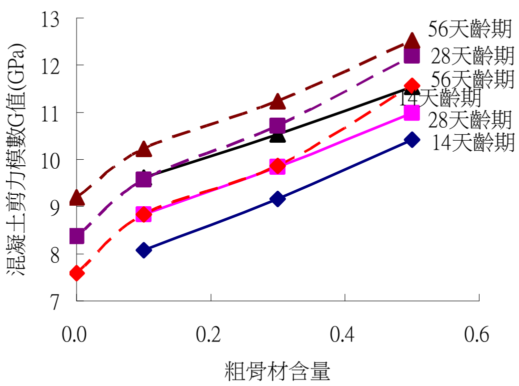

Fig. 4 Elastic shear modulus of concrete.

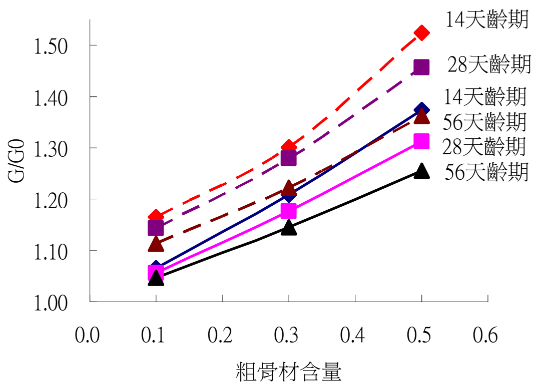

Fig. 5 Increasing ratio of shear modulus.

To the end, we compare the Young modulus calculated from ACI 318-89 or Eq. (1) to M-T moduli and the experiments. Fig. 6, 7 and 8 are the Young modulus at aged 14, 28 and 56 days respectively. The upper bounds and lower bounds of Young modulus are determined under a prescribed displacement and prescribed traction respectively as follows.

$$E = c_0 E_0 + c_1 E_1 \tag{9}$$

$$E = \frac{E_0 E_1}{c_0 E_1 + c_1 E_0} \tag{10}$$

Apparently, the Young modulus calculated from ACI Code is overestimated when c_1 is less than 30% depicted in Figs. 6-8. However, the estimations from the M-T approach are close to the experiments. This means that the Young modulus determined from M-T method is better than that from ACI Code.

5. Conclusions

After we compare the elastic moduli of concrete obtained from the experiments, the micromechanics approach and ACI Code, and find the following results.

(1) The M-T moduli of concrete with the spherical inclusions are always lower than those ones in experiments.

(2) The elastic Young and shear moduli of concrete calculated from M-T approach are the pretty reliable estimations by comparing to the experimental results, especially at increasing the volume concentration of aggregates and the longer ages of concrete.

(3) When the volume fraction of coarse aggregates reaches c_1 =50%, the M-T shear modulus of concrete vary from the experiments.

(4) The elastic Young modulus of concrete determined from ACI Code is obviously overestimated, and will close to the experiments when the total volume concentration of the fine and coarse aggregate reaches 70% or after.

(5) The elastic moduli of concrete do depend on the volume fraction of the aggregate, and the material properties of the constituents. The predicted formula based on those factors may be the better way to estimate

the elastic moduli of concrete rather than ACI Code, especially the total volume fraction of aggregates less that 70%.

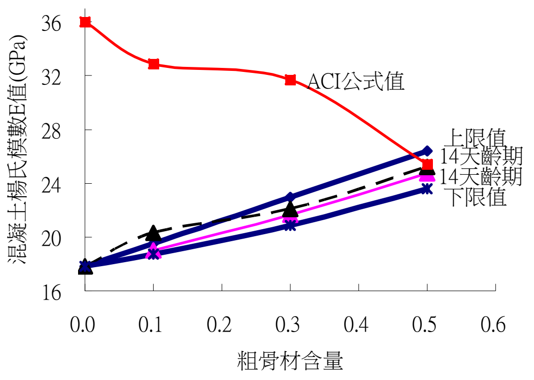

Fig. 6 Comparisons of Young modulus at aged 14 days.

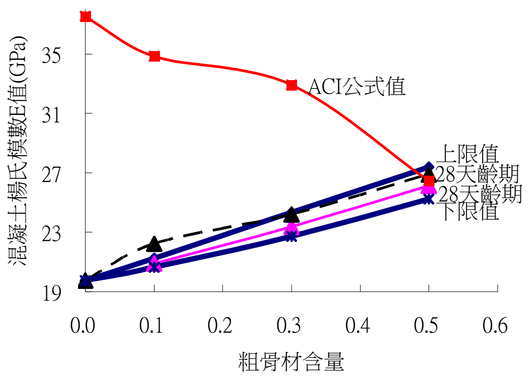

Fig. 7 Comparisons of Young modulus at aged 28 days.

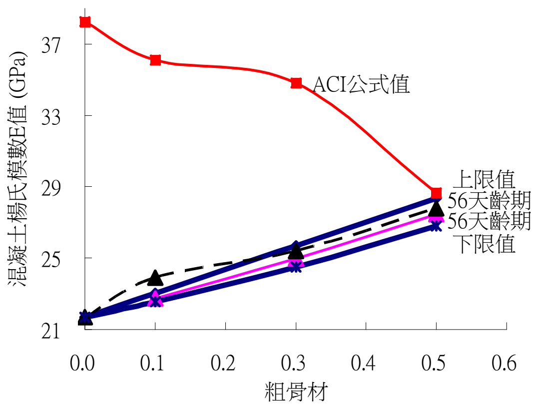

Fig. 8 Comparisons of Young modulus at aged 56 days.

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Appendix A: Material Constants

Without any summation over any repeated indices and with *i*, *j*, *k*, always following the 1, 2, 3 even permutation, the constants are

$$p_1 = 1 + c_1[(a_{11} + a_{12} + a_{13})(S_{3311} + S_{2211} + S_{1111} - 1) + (a_{21} + a_{22} + a_{23})(S_{3322} + S_{2222} + S_{1122} - 1) + (a_{31} + a_{32} + a_{33})(S_{3333} + S_{2233} + S_{1133} - 1)]/3,$$

$$p_2 = (a_{11} + a_{12} + a_{13} + a_{21} + a_{22} + a_{23} + a_{31} + a_{32} + a_{33})/3,$$

$$b_1 = a_{11}(S_{1111} - 1) + a_{21}S_{1122} + a_{31}S_{1133},$$

$$b_2 = [(a_{12} + a_{13})(S_{1111} - 1) + (a_{22} + a_{23})S_{1122} + (a_{32} + a_{33})S_{1133}]/2,$$

$$b_3 = [a_{11}(S_{2211} + S_{3311}) + a_{21}(S_{2222} + S_{3322} - 1) + a_{31}(S_{3333} + S_{2233} - 1)]/2,$$

$$b_4 = [(3a_{33} + a_{32})(S_{3333} - 1) + (3a_{23} + a_{22})S_{3322}$$

$$+(3a_{13}+a_{12})S_{3311}+(3a_{32}+a_{33})S_{2233}$$
$$+(3a_{22}+a_{23})(S_{2222}-1)+(3a_{12}+a_{13})S_{2211}$$
$$+2b_{23}(2S_{2323}-1)]/8,$$

$$b_5=[(3a_{32}+a_{33})(S_{3333}-1)+(3a_{22}+a_{23})S_{3322}$$
$$+(3a_{12}+a_{13})S_{3311}+(3a_{33}+a_{32})S_{2233}$$
$$+(3a_{23}+a_{22})(S_{2222}-1)+(3a_{13}+a_{12})S_{2211}$$
$$-2b_{23}(2S_{2323}-1)]/8,$$

$$b_6=[b_{12}(2S_{1212}-1)+b_{13}(2S_{1313}-1)]/2,$$

$$b_{12}=(1-\mu_1/\mu_0)/[1+2S_{1212}(\mu_1/\mu_0-1)],$$

$$b_{13}=(1-\mu_1/\mu_0)/[1+2S_{1313}(\mu_1/\mu_0-1)],$$

$$b_{23}=(1-\mu_1/\mu_0)/[1+2S_{2323}(\mu_1/\mu_0-1)],$$

$$a_{ii}=[3(\kappa_1-\kappa_0)(\mu_1-\mu_0)^2(S_{jjjj}S_{kkkk}-S_{jjkk}S_{kkjj})$$
$$-(\mu_1-\mu_0)(\kappa_1\mu_0-\kappa_0\mu_1)(S_{jjjj}+S_{kkkk}-S_{jjkk}-S_{kkjj})$$
$$+3\mu_0(\kappa_1-\kappa_0)(\mu_1-\mu_0)(S_{jjjj}+S_{kkkk})$$
$$+3\kappa_0\mu_0(\mu_1-\mu_0)+\mu_0(\kappa_1\mu_0-\kappa_0\mu_1)]/A,$$

$$a_{ij}=[3(\kappa_1-\kappa_0)(\mu_1-\mu_0)^2(S_{iikk}S_{kkjj}-S_{iijj}S_{kkkk})$$
$$-(\mu_1-\mu_0)(\kappa_1\mu_0-\kappa_0\mu_1)(S_{iikk}+S_{kkjj}-S_{iijj}-S_{kkkk})$$
$$-3\mu_0(\kappa_1-\kappa_0)(\mu_1-\mu_0)S_{iijj}+\mu_0(\kappa_1\mu_0-\kappa_0\mu_1)]/A,$$

$$A=(\mu_1-\mu_0)(\kappa_1\mu_0-\kappa_0\mu_1)[S_{3333}(S_{1111}+S_{2222}$$
$$-S_{1122}-S_{2211})+S_{3322}(S_{1133}+S_{2211}-S_{1111}-S_{2233})$$
$$+S_{3311}(S_{1122}+S_{2233}-S_{1133}-S_{2222})$$
$$+S_{2211}(S_{1133}-S_{1122})+S_{2222}(S_{1111}-S_{1133})$$
$$+S_{2233}(S_{1122}-S_{1111})]+3(\kappa_1-\kappa_0)\cdot$$
$$(\mu_1-\mu_0)^2[S_{3333}(S_{1122}S_{2211}-S_{1111}S_{2222})$$
$$+S_{3322}(S_{1111}S_{2233}-S_{1133}S_{2211})+S_{3311}\cdot$$
$$(S_{1133}S_{2222}-S_{1122}S_{2233})]+3\mu_0(\kappa_1-\kappa_0)\cdot$$
$$(\mu_1-\mu_0)(S_{1122}S_{2211}+S_{1133}S_{3311}+S_{2233}S_{3322}$$
$$-S_{1111}S_{2222}-S_{2222}S_{3333}-S_{3333}S_{1111})$$
$$-\mu_0(\kappa_1\mu_0-\kappa_0\mu_1)(S_{1111}+S_{1122}+S_{1133}+S_{2211}$$
$$+S_{2222}+S_{2233}+S_{3311}+S_{3322}+S_{3333})-3\kappa_0\mu_0\cdot$$
$$(\mu_1-\mu_0)(S_{1111}+S_{2222}+S_{3333}-1)-3\kappa_0\mu_0\mu_1,$$

Appendix B: Eshelby's S_{ijkl} Tensor

Eshelby's tensor is in terms of the Poisson ratio of the matrix ν_0 and the shape of the inclusion. It has a property of $S_{ijkl}=S_{jikl}=S_{ijlk}$. The three radii of an ellipsoid are a_1, a_2 and a_3, where $a_1>a_2>a_3$. The properties of an ellipsoid within the matrix can be described by twelve independent components of the Eshelby tensor, which are

$$S_{1111}=[3(a_1I_{11})+(1-2\nu_0)I_1]/[8\pi(1-\nu_0)]$$
$$S_{1122}=[a_2^2I_{12}-(1-2\nu_0)I_1]/[8\pi(1-\nu_0)]$$
$$S_{1133}=[a_3^2I_{13}-(1-2\nu_0)I_1]/[8\pi(1-\nu_0)]$$
$$S_{1212}=[(a_1^2+a_2^2)I_{12}+(1-2\nu_0)(I_1+I_2)]/[16\pi(1-\nu_0)]$$

and all other non-zero components are obtained by the cyclic permutation of (1, 2, 3). The S_{ijkl} tensor which can not be calculated from above are zero. The I_i and I_{ij} components are

$$I_1=(4\pi a_1a_2a_3)[F(\theta,k)-E(\theta,k)]$$
$$/[(a_1^2-a_2^2)(a_1^2-a_3^2)^{1/2},$$

$$I_3=(4\pi a_1a_2a_3)[a_2(a_1^2-a_3^2)^{1/2}/(a_1a_3)-E(\theta,k)]$$
$$/[(a_2^2-a_3^2)(a_1^2-a_3^2)^{1/2},$$

$$I_2=4\pi-I_1-I_3\quad,$$

$$I_{11}=[4\pi a_2^2(a_1^2-a_3^2)+a_1^2(a_2^2+2a_3^2-3a_1^2)I_1$$
$$+a_1^2(a_3^2-a_2^2)I_3]/[3a_1^2(a_1^2-a_2^2)(a_3^2-a_1^2)],$$

$$I_{22}=4\pi\{[a_1^2a_3^2-a_2^2(2a_1^2+2a_3^2-3a_2^2)]$$
$$+a_2^2(a_1^2+2a_3^2-3a_2^2)I_1+a_2^2(a_3^2+2a_1^2-3a_2^2)I_3\}$$
$$/[3a_2^2(a_1^2-a_2^2)(a_3^2-a_2^2)],$$

$$I_{33}=[4\pi a_2^2(a_3^2-a_1^2)+a_3^2(a_1^2-a_2^2)I_1$$
$$+a_3^2(a_2^2+2a_1^2-3a_3^2)I_3]/[3a_3^2(a_1^2-a_3^2)(a_3^2-a_2^2)]$$

$$I_{12}=(4\pi-2I_1-I_3)/(a_1^2-a_2^2),$$

$$I_{13}=(I_3-I_1)/(a_1^2-a_3^2),$$

$$I_{23}=(I_1+2I_3-4\pi)/(a_2^2-a_3^2)$$

and the elliptic integrals of the first and the second kind are

$$F(\theta,k)=\int_0^\theta[1/(1-k^2\sin^2w)^{1/2}]\cdot dw,$$

$$E(\theta,k)=\int_0^\theta(1-k^2\sin^2w)^{1/2}\cdot dw$$

where

$$\theta=\sin^{-1}(1-a_3^2/a_1^2)^{1/2},$$

$$k=[(a_1^2-a_2^2)/(a_1^2-a_3^2)]^{1/2}.$$

E/E_0