LONG-TERM STRESS-STRAIN RELATIONS OF THE CEMENT-MATRIX COMPOSITE

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Summary Based on a micromechanics-based theory and Burgers' rheological model, a stress-strain relationship depending on the material ages, not the creep, of the cement-matrix composites is determined. Unlike traditional models of the cement-matrix composite, the proposed micromechanics model has the predictive power depending on the material ages for the stress-strain behaviour of a mortar that carries different from amount of aggregates.

INTRODUCTION

Traditional models, based on phenomenological approach, have been proposed to estimate the stress-strain behaviour of the cement-matrix composite such as mortar or concrete [1-2]. Most of them set the influence factors by means of the properties of the composite. The composite properties were not derived from the properties of the individual phases but rather by inverse fitting of the measures data. Thus the simulated stress-strain relations of the composite material are difficult to reproduce as the aggregate content or testing conditions changes, as in that case the material constants must be fitted again for a new composite system. Since concrete consists of the cement paste (or the binders) and the aggregates, its properties are highly dependent on the individual properties and volume concentration, and shape, of the aggregates. The approach here is developed so that it can be applied to conditions of different constitutions without having to determine the material properties of the composite for each and every case.

One takes concrete as a composite with cement-based binders as the matrix and the aggregates as reinforcing inclusions. One also takes the nonlinear response of the binders and the volume concentration of the aggregates as the two dominant factors in determining the overall response of the cement-based composites. The inclusions of the same shape but different sizes are taken to perfectly bonded to the matrix. The results of this assumption will coincide with those of the Double-inclusion model when the double-cells have the same shape and orientation as the enclosed inclusions [3]. The matrix phase here is to be broadly viewed as a combination of cement paste, fly ash, slag, silica fume, and other admixtures. The composite material as a whole then is a two-phase composite with an overall isotropy. Moreover the inclusions are taken to be homogeneously dispersed in the matrix, with an average shape grossly represented by a spheroidal inclusion with an aspect ratio α (the length-to-diameter ratio). In the two-phase system, the matrix will be referred to as phase 0, and the inclusions as phase 1. The volume concentration of the r-th phase will be denoted by c_r $(c_1 + c_0 = 1)$, and the bulk and shear moduli of the r-th phase will be written as κ_r and μ_r, respectively. Here one shall focus on the behavior of mortar at various concentrations of aggregate and the material ages.

MECHANICAL MODEL OF THE CEMENT-BASED BINDERS

Cement paste is investigated and tested when the ratio of water/cement was 0.485 and specimens were under a uniaxial compression with a constant strain rate 1×10^{-5}/sec. One finds that the nonlinear stress-strain curve of the binders can be represented by Burgers' four-parameter rheological model with two springs and two dashpots, as depicted in Fig. 1. The stress-strain relation of the cement-based binders has been solved as

$$\sigma(\varepsilon,t) = 3.71 f_u(t) \times [e^{m_1\varepsilon\times10^3} - 1.009 e^{m_2\varepsilon\times10^3}] + 0.0334 f_u(t) \quad (1)$$

where m_1(t) and m_2(t) are the characteristic roots satisfying the governing differential equation for this four-parameter model, the time t is the material age of the binders, $\sigma(\varepsilon,t)$ the stress at a given stage of strain ε, and $f_u(t)$ the peak stress, respectively. Meanwhile, the stress-strain relation in Eq. (1) is also found and displayed a satisfactory agreement for the mortar. This shows that the Burgers model can describe the nonlinear curves for both types of materials, but for the mortar its constants will have to be determined by simulation for each and every case as its aggregate content changes, that is, its constants cannot be derived from those of the cement paste. Despite the good simulation such a curve-fitting procedure for the mortar has no predictive power and is what we have tried to avoid from the outset. This is what has prompted us to develop a micromechanics-based secant moduli method so that the nonlinear behaviour of mortar can be calculated from those of its cement paste at every volume fraction of the aggregates, c_1.

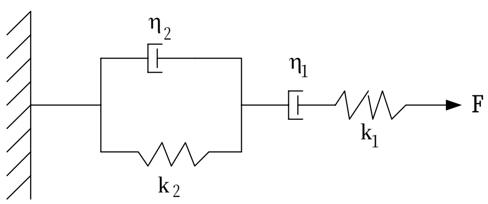

Fig. 1 The four-parameter model on cement-based binders.

THE STRESS-STRAIN CURVES OF CEMENT-MATRIX COMPOSITES

The effective elastic moduli of a two-phase composite, based on the Eshelby-Mori-Tanaka theory, have been shown to intimately relate to the universal Hashin-Shtrikman bounds for the isotropic case. The explicit forms of the effective elastic moduli with ellipsoidal inclusions are also established, and cast into

$$\kappa = \kappa_0 /(1 + c_1 p) \ ; \mu = \mu_0 /(1 + c_1 q) \tag{2}$$

where the material constants p and q are in terms of the volume concentration c_1 and shape of inclusions as represented by Eshelby's S-tensor, and the elastic bulk and shear moduli of both constituent phases.

The stress-strain relations of cement paste or the cement-based binders are nonlinear although its fracture behaviour is brittle. For the calculation of nonlinear stress-strain relations of the composite, one extends the elastic moduli in Eq. (2) to the secant moduli for the cement paste and the composite. This is the approach by means of a linear comparison matrix whose elastic moduli are taken to be identical to the secant moduli of the nonlinear matrix at a given stage of deformation. The secant bulk and shear moduli, κ_0^s and μ_0^s, and the secant Young modulus E_0^s of the binders are from the isotropic connections with the Poisson ratio ν_0 being assumed to remain constant. The secant Young modulus of the cement paste at a given stage of deformation can be determined from Eq. (1) by

$$E_0^s(\varepsilon, t) = \sigma(\varepsilon, t) / \varepsilon \tag{3}$$

The overall effective secant moduli of the composite then can be determined by replacing κ_0 and μ_0 by κ_0^s and μ_0^s, respectively. Under a uniaxial compression, the stress-strain curve of the cement-matrix composite follows as $\bar{\sigma}(\varepsilon, t) = E^s(\varepsilon, t) \cdot \bar{\varepsilon}$, which provides the desired overall stress versus strain relations.

RESULTS AND DISCUSSION

In order to evaluate the validity of the proposed approach, one examines the stress-strain curve of mortar with the same material properties of cement paste at the 7^{th} and 28^{th} day of the ages, respectively. The shape of aggregates is approximately $\alpha = 1.13$. Three mortars with c_1 = 0.5, 0.6 and 0.7 are plotted in Fig. 2-3. Among those figures, the solid lines are theoretically predicted and the symbols are the experimental data, where cement paste with c_1 = 0.0 actually is simulated by the Burgers model or Eq. (1) at different material ages. It is observed that the predicted curves are in close agreement with the test data especially the stress-strain relations below 80% the peak strength, beyond which the microcrack link-up will dominate the stress-strain behaviour. Meanwhile, the developed approach can show the stress-strain relations of the cement-matrix composite affected by the material ages and the volume concentrations of the aggregate.

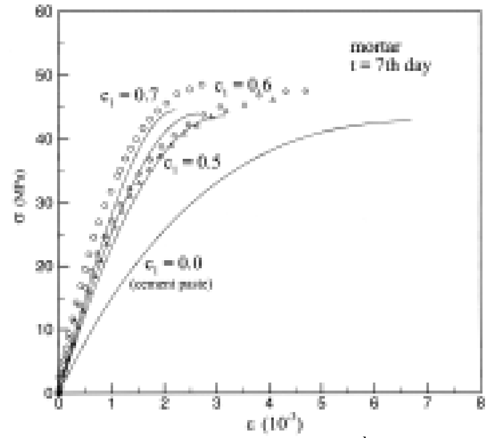

Fig.2 Stress-strain curves at the 7^{th} aged day.

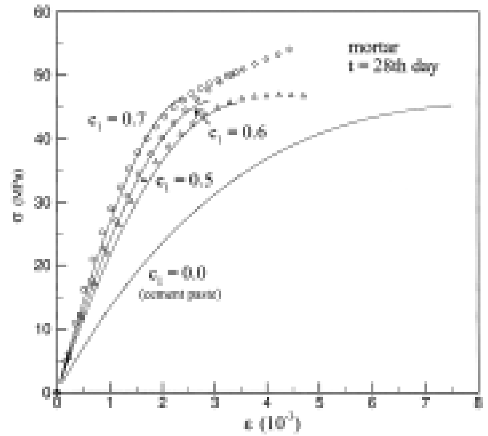

Fig.3 Stress-strain curves at the 28^{th} aged day.

CONCLUSIONS

The proposed theory is micromechanics-based, and is established for the overall nonlinear stress-strain behaviour of the cement-matrix composite in terms of the volume concentrations and the material ages. The predicted stress-strain relations are suitable for the volume concentration up to 70% of the aggregates and the range lower than 80% of the peak strength.

References

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