Chapter 9 Managing Inventory

- Types of Inventory
- ABC Analysis
- Q models and P models
- Supplement C: Special Inventory Models

Inventory Management at NETFLIX

- 44M streaming customers worldwide and 7M DVD customers in US.
- 89M discs distributed across 39 warehouses in US. Each can process 60000 orders per day.
- Discs are scanned by machines to see if any customer has ordered them. Others are shelved for future rentals.
- Proper managed inventories can support competitive priorities of variety and delivery speed.

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What is Inventory?

A stock of materials used to satisfy customer demand or to support the production of services or goods.

Manufacturing Inventory

raw materials, WIP, maintenance and repairs, supplies, FGI

Retail Inventory

merchandise, supplies 餐飲業？

What is a Inventory Management?

The planning and controlling of inventories to meet the competitive priorities of the organization.

Video: What Is Inventory Management

Pressures for Small Inventories

Inventory holding cost

- Cost of capital (opportunity cost)
- Storage and handling costs: space, labor, machines
- Taxes 各國稅法不同, Insurance
- Shrinkage
  - Pilferage by customers or employees
  - Obsolescence due to model changes or new products
  - Deterioration: limited shelf life

Pressures for Large Inventories

- Customer service: stockout (lost sale) or backorder
- Ordering cost: fixed cost of preparing a purchase order
- Transportation cost: full truckload or full container
- Payments to suppliers (quantity discount) 採購↔倉管
- Setup cost: fixed cost of changing over a machine to produce a different product.
- Labor and equipment utilization

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Inventory Costs

- Purchase Cost
- Holding or Carrying cost ← order too much or too early
  opportunity cost 資金積壓的潛在損失
  storage cost 倉儲設備、進出盤點
  shrinkage cost 貶值、偷竊、毀損、保費
- Ordering or Setup cost 前置作業成本 ← order too often or too little
  對外採購：聯繫、運輸、驗收　　內部製造：停機調整、試產
- Shortage costs or Lost Sales ← order too little or too late
  停工減產的損失、延誤交貨的罰款
  銷售減少的利潤損失

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Type of Inventory and Reduction Tactics

- Cycle inventory 訂貨間隔的正常存貨: Reduce the lot size
  - Reduce ordering and setup costs and allow *Q* to be reduced
- Safety stock inventory 預測需求以外的安全庫存: Place orders closer to the time when they must be received
  - Improve demand forecasts
  - Reduce lead times and supply uncertainties
- Anticipation inventory 旺季前製造庫存或漲價前大量進貨: Match demand rate with production rates
  - Add new products with different demand cycles 季節互補
  - Provide off-season promotional campaigns
- Pipeline inventory 在途庫存: Reduce lead times
  - Find more responsive suppliers and select new carriers

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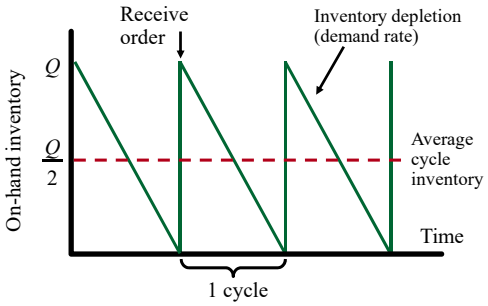
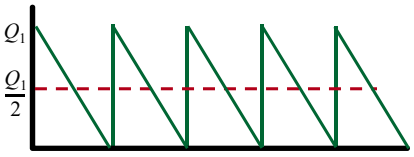
Economic Order Quantity 不須預測需求



How Many to Order?



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Calculating EOQ

- Annual Demand = D
- Annual holding cost
  = (Average cycle inventory) × (Unit holding cost) = $\frac{Q}{2} \times H$
- Annual ordering cost
  =(Number of orders/Year) × (Ordering or setup costs) = $\frac{D}{Q} \times S$

Total costs = $C(Q) = \frac{Q}{2} H \times 1 + \frac{D}{Q} S$

$\frac{d}{dQ} C = 0 \Rightarrow \text{EOQ} = Q^* = \sqrt{\frac{2DS}{H}}$ Time Between Orders = $\frac{\text{EOQ}}{D}$

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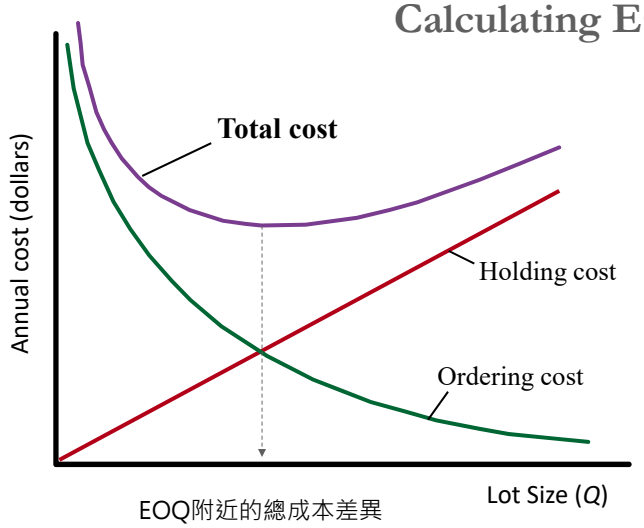
Calculating EOQ



EOQ附近的總成本差異

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Example 9.2: Bird Feeder

- Sales are 18 units per week, the supplier charges $60 per unit.
- Ordering cost is $45.
- Annual holding cost is 25 percent of a feeder's value.
- What is the annual cycle-inventory cost of the current policy of using a 390-unit lot size? Would a lost size of 468 be better?

D=(18 units/week)(52 weeks/year) = 936 units

H=0.25($60/unit) = $15 S=$45

$$C(Q) = \frac{Q}{2}H \times 1 + \frac{D}{Q}S = \frac{390}{2}(\$15) + \frac{936}{390}(\$45)$$

$$= \$2{,}925 + \$108 = \$3{,}033$$

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Example 9.3 EOQ for bird feeders

$$\text{EOQ} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(936)45}{15}} = \textbf{74.94 or 75 units}$$

Parameters

Current Lot Size (Q)	390
Demand (D)	936
Order Cost (S)	$45
Unit Holding Cost (H)	$15

Economic Order Quantity	75

Annual Costs

Orders per Year	2.4
Annual Ordering Cost	$108.00
Annual Holding Cost	$2,925.00
Annual Inventory Cost	$3,033.00

Annual Costs based on EOQ

Orders per Year	12.48
Annual Ordering Cost	$561.60
Annual Holding Cost	$562.50
Annual Inventory Cost	$1,124.10

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Inventory Control Systems

Periodic Review System

Fixed Order Period
P model

Continuous Review System

Fixed Order Quantity
Q model

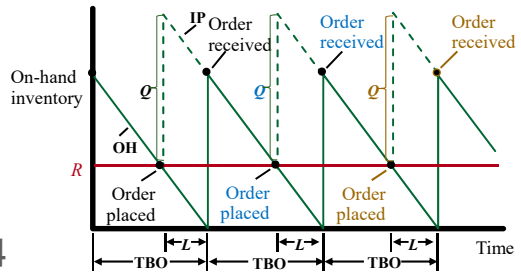
Continuous Review System 持續監控庫存

- Q model or fixed order quantity system
  - Tracks inventory position (IP)
  - Reorder point system (ROP) and fixed order quantity (Q)
  - Decision Includes scheduled receipts (SR), on-hand inventory (OH), and back orders (BO)

$$\text{Inventory position (IP)} = \underset{\text{存貨水準}}{} \text{OH} + \underset{\text{已訂未到}}{\text{SR}} - \underset{\text{欠貨}}{\text{BO}}$$
現有

Rule: If IP ≤ ROP, place an order of size Q

Example 9.4

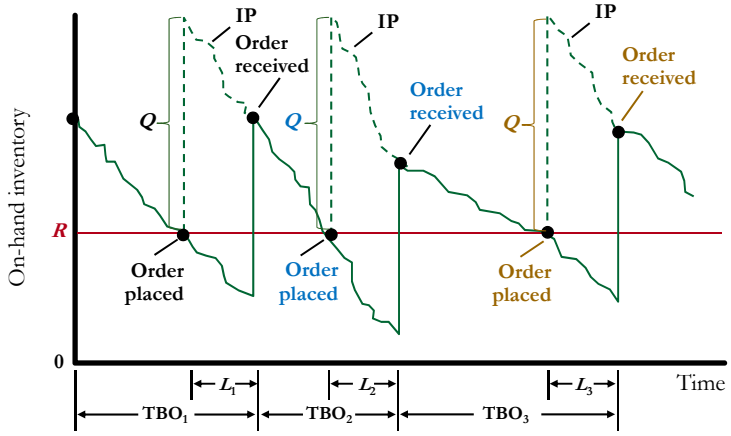


Demand for chicken soup at a supermarket is always 25 cases a day and the lead time is always 4 days. The on-hand inventory has only 10 cases. No backorders currently exist, but there is one open order in the pipeline for 200 cases. Should a new order be placed?

R = Total demand during lead time = (25)(4) = 100 cases

IP = OH + SR – BO= 10 + 200 – 0 = 210 cases

Continuous Review System 2: Variable Demand



Select ROP base on Demand, Lead Times, and Safety Stock.

Example 9.5 ROP=300, Q=250, L=5 days

Day	Demand	OH	SR	BO	IP	Q
1		400			400 + 0 = 400	
2	60	340			340 + 0 = 340	
3	80	260	250 after ordering		260 < *R* before ordering 260+250=510 after ordering	250 due Day 8
4	40	220	250		220 + 250 = 470	
5	75	145	250		145 + 250 = 395	
6	55	90	250		90 + 250 = 340	
7	95	0	250+ 250 = 500 after ordering	5	0+250–5=245<*R* before ordering 245 + 250 = 495 after ordering	250 due Day 12
8	50	250-50-5 =195	250		195 + 250 = 445	

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Continuous Review System 2: Reorder Point

- Constant lead time and variable demand 前置期間內的需求不確定

Reorder point = Average demand during lead time + Safety stock

$= \bar{d}L$ + Safety stock

額外保留的商品量以避免缺貨

- Choosing a Reorder Point
  2. Determine the distribution of demand during lead time
  1. Choose an appropriate service-level policy 對商品不缺貨的要求
  3. Use the formula to calculate the safety stock and reorder point levels

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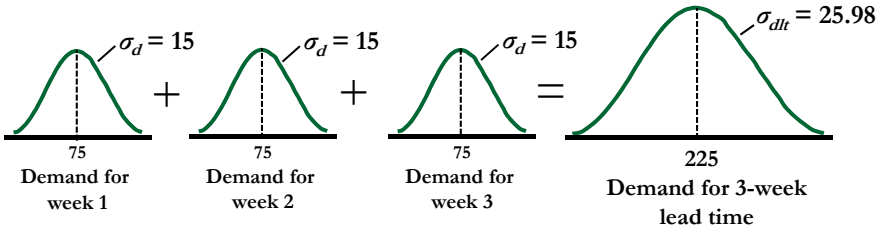
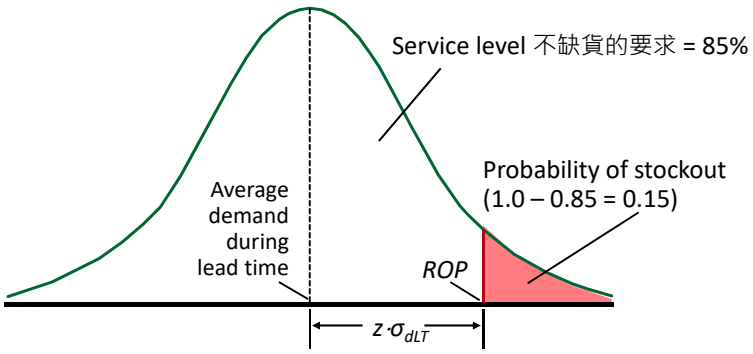
Distribution of Demand during Lead Time

- Specify mean and standard deviation of daily demand 預測
  $\bar{d}$ σ_d
  - Standard deviation of demand during lead time

$$\sigma_{dLT} = \sqrt{\sigma_d^2 L} = \sigma_d \sqrt{L}$$ 交貨時間越長...



Continuous Review System 2: Service Level



Service level = 85% ⇒ $z = \Phi(0.85)$ = NORMINV(0.85,0,1)=1.036

Continuous Review System 2 : Safety Stock

Safety stock = $z \cdot \sigma_{dLT} = z \cdot \sigma_d \sqrt{L}$ 3個影響因素

z =number of standard deviations needed to achieve the service level
σ_d =stand deviation of daily demand
σ_{dLT} =stand deviation of demand during lead time

If $\bar{d}$=75, σ_d =15, L=3 and service level=85%,
then safety stock = $1.036 \cdot 15\sqrt{3}$ = 26.92

Reorder point = $R = \bar{d}L$ + safety stock = (75)3+26.92

If service level=95%, safety stock = $1.645 \cdot 15\sqrt{3}$

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Continuous Review System 3

- Both Demand and Lead Time are Variable 如果交期也不確定

ROP = (Average daily demand × Average lead time) + Safety stock
= $\bar{d}\,\bar{L}$ + Safety stock = $\bar{d}\,\bar{L} + z\sigma_{dLT}$

where
$\bar{d}$ =average daily (or weekly or monthly) demand
$\bar{L}$ =average lead time (相同時間單位)
$\sigma_{dLT} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$

σ_d =standard deviation of daily demand
σ_{LT} =standard deviation of the lead time

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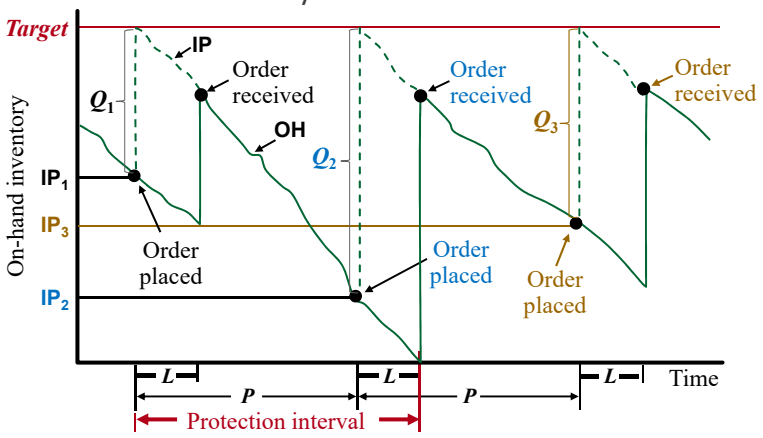
Periodic Review System 固定週期管理，不管存貨多寡

- Fixed interval reorder system or P model



Rule: Check inventory (IP) and then place order to meet target

Example 9.8 Target = 620, P = 6 days, L = 5 days

Day	Demand	OH	SR	BO	IP	Q
1		400			400	
2	60	340	280 after ordering		340 before ordering 340+280=620 after ordering	620 – 340 = 280 due Day 7
3	80	260	280		260 + 280 = 540	
4	40	220	280		220 + 280 = 500	
5	75	145	280		145 + 280 = 425	
6	55	90	280		90 + 280 = 370	
7	95	90+280–95=275			275 + 0 = 275	
8	50	225	395 after ordering		225 before ordering 225+395=620 after ordering	620–225=395 due Day 13

Periodic Review: variable demand & fixed lead time

- Selecting the time between reviews 檢查週期 $P = \frac{EOQ}{\bar{d}}$
- Set Target Inventory Level. 目標庫存上限 T must cover demand over a protection interval of $P + L$
  - Average demand during the protection interval is $\bar{d}(P + L)$ 預測
  - Safety stock for the protection interval = $z \cdot \sigma_{P+L} = z \cdot \sigma_d \sqrt{P + L}$
  - z is determined by the service level.
  - $T = \bar{d}(P + L) + z \cdot \sigma_d \sqrt{P + L}$
- Order Quantity Q = T – Inventory Position (IP)

  當期訂購量　目標庫存上限　庫存水準

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Example 9.9: Bird Feeder

- Weekly demand for the bird feeder is normally distributed with a mean of 18 units and a standard deviation of 5 units.
- The lead time is 2 weeks. The Q system called for an EOQ of 75 units and a safety stock of 9 units for a cycle-service level of 90%.
- What is the equivalent P system?

$$P = \frac{EOQ}{\bar{d}} = \frac{75}{18} \approx 4 \text{ weeks} \qquad \text{Service level of } 90\% \Rightarrow z=1.28$$

$$T = \bar{d}(P + L) + z \cdot \sigma_d \sqrt{P + L} = 18\,(4+2) + 1.28\,(5)\sqrt{4 + 2}$$
$$= 108+15.68 \approx 124$$

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Costs for Q and P Systems

- Total Annual *Q* System Costs

$$TC = \frac{Q}{2}(H) + \frac{D}{Q}(S) + (\text{Safety stock})\,(H)$$

- Total Annual *P* System Costs

$$= \frac{\bar{d}P}{2}H + \frac{D}{\bar{d}P}S + (\text{safety stock})H$$

- The P system requires more inventory for the same level of protection against stockouts or backorders.
- Optional Replenishment System 選擇補貨系統

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ABC庫存分類管理 Pareto's 80/20 principle

SKU no.	annual demand	unit cost	annual dollar value
1	2500	330	**825000**
2	1000	70	70000
3	1900	500	**950000**
4	1500	100	150000
5	3900	700	**2730000**
6	1000	915	**915000**
7	200	210	42000
8	1000	4000	**4000000**
9	8000	10	80000
10	9000	2	18000
11	500	200	100000
12	400	300	120000

There are other ways to do ABC classification. Review ABC classification periodically.

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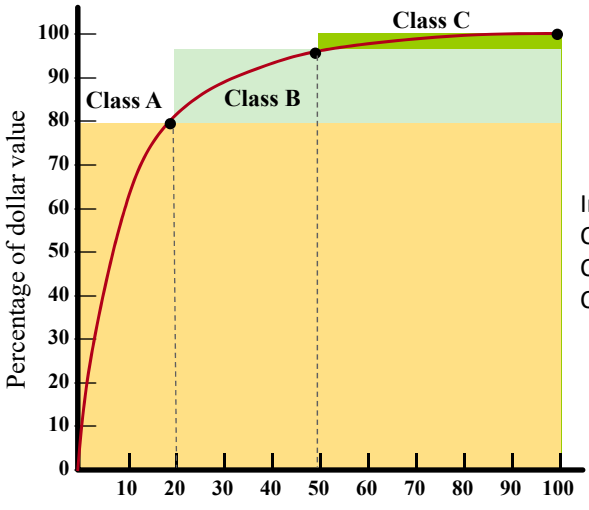
Pareto Chart for ABC Analysis



Inventory Accuracy by APICS
Class A: ±0.2%
Class B: ±1%
Class C: ±5%

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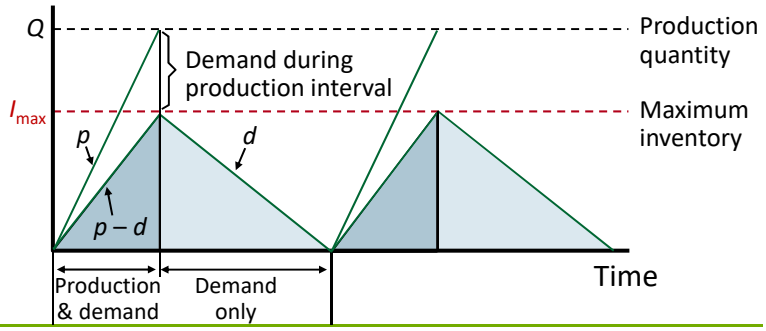
Supplement C: Economic Production Quantity

- 經濟生產批量Production Quantity *Q*.
- Production rate p > the demand rate d, so there is a buildup of $(p - d)$ units per time period.
- Buildup continues for Q/p periods.



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Economic Production Quantity economic lot size

Max. inventory is: $I_{max} = (p-d) \times \text{production time} = (p-d)\frac{Q}{p}$

Average inventory is no longer $Q/2$, it is $I_{\max}/2$

Total annual cost = Annual holding cost + Annual setup cost

$$C = \frac{I_{\max}}{2}(H) + \frac{D}{Q}(S) = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S)$$

成本最小化 $\Rightarrow EPQ = ELS = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-d}}$

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Example C.1

A plant manager must decide the lot size for a product that has a steady demand of 30 units per day. The production rate is 190 units per day, annual demand is 10,500 units, setup cost is $200, annual holding cost is $0.21 per units, and the plant operates 350 days per year.

$$EPQ = \sqrt{\frac{2DS}{H}}\sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(10500)(200)}{0.21}}\sqrt{\frac{190}{190-30}} = 4873.4$$

$$C = \frac{Q}{2}\left(\frac{p-d}{p}\right)(H) + \frac{D}{Q}(S) = \frac{4873.4}{2}\left(\frac{190-30}{190}\right)(0.21) + \frac{10500}{4873.4}(200)$$

Q: What are the advantages of reducing the setup time by 10%?

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Supplement C: One-Period Decisions

- 適用於不可庫存或庫存價值低之商品採購與製造、旅遊業超額訂位
- Example C.3: A gift shop sells a Christmas ornament carved from wood and makes a $10 profit per unit sold during the season, but it takes a $5 loss per unit after the season is over.

Step 1: List the demand levels and probabilities.

Demand	10	20	30	40	50
Demand Probability	0.2	0.3	0.3	0.1	0.1

預測

Step 2: Develop a payoff table that shows the profit for each purchase quantity, *Q*, at each assumed demand level, *D*.

One-Period Decisions

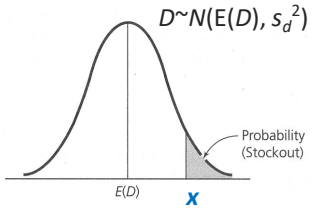
Payoff = (Profit per unit)(Demand quantity) – (Loss per unit)(Leftovers)

$$= \begin{cases} p \cdot Q & \text{if Demand} \geq \text{Order Quantity} \\ p \cdot D - l\,(Q - D) & \text{if Demand} < \text{Order Quantity} \end{cases}$$

		Demand					Expected Payoff
		10 (0.2)	20 (0.3)	30 (0.3)	40 (0.1)	50 (0.1)	
Order Quantity	10	100	100	100	100	100	**100**
	20	50	200	200	200	200	**170**
	30	0	150	300	300	300	**195** ←
	40	-50	100	250	400	400	**175**
	50	-100	50	200	350	500	**140**

Steps 3 &4: Calculate the expected payoff of each Q and pick the best.

Newsboy Problem



預測報紙需求D為常態分布
平均值=90 papers　標準差=10 papers

進貨90 papers ⇒ P(stockout)=50%

C_e=進貨高估需求的單位成本=進價-殘值　報紙進價=0.20

C_s=進貨低估需求的單位成本=售價-進價　報紙利潤=0.30

目前進貨90份，增加進貨是否能增加利潤？

P=P(增加的進貨賣不出)=0.5 ⇒ (1-P) ·C_s=0.5(0.30) > P·C_e=0.5(0.20)
潛在利潤　潛在損失

Key: 增加進貨量直到 $P \approx \frac{C_s}{C_e + C_s}$ ← critical ratio or optimal service level

Airline Overbooking (Revenue Management)

飛機有90個座位，經常有人訂位卻臨時取消或延期
假設 probability of no-show=10%，訂位額滿時預期會有9個空位
Solution: 超額訂位

Decision: 接受100個訂位 ⇒ no-shows ~$N(10, s^2)$
⇒ P(no-shows<10)=50% ⇒ P(overbooking)=50%

訂位系統高估 no-show (座位不足)的單位成本=C_e
低估 no-show (旅客未報到)的單位成本=C_s

P(no overbooking)>80% ⇒ 減少超額訂位

Q: 如果 C_s << C_e，航空公司應如何調整超額訂位？